TIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

JUN 13 1922
MAILED

o: M. Truscott

TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

---

No. 100.

---

# THEORY OF THE SLOTTED WING.

Lecture by A. Betz, Göttingen.

---

June, 1922.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

# ERRATA

## TECHNICAL NOTE No. 100.

## THEORY OF THE SLOTTED WING

Lecture by A. Betz, Göttingen.

The following corrections have been made in Technical Note No. 100, entitled "Theory of the Slotted Wing." Please insert this sheet in your copy of the report.

Page 6.

Second paragraph, fourth sentence should read: We also know that, at this point the flow is obliquely upward.

Page 8.

Third paragraph, fifth sentence should read: Thus we obtain here, near the leading edge and mainly on top, a decrease in pressure.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL NOTE NO. 100.

# THEORY OF THE SLOTTED WING.*

Lecture by A. Betz, Göttingen.

Through the intensive study of all technical aviation problems during the war, the most important airplane parts, especially the wing, were so thoroughly tested as to create the impression that no further substantial improvement was possible. The characteristics of the different wing sections were sufficiently known to enable one to select the most suitable section for almost any purpose.

Then the discovery by Lachmann and Handley-Page suddenly revealed entirely new possibilities and the wing section again became a rich field of problems. As probably you all know, this discovery consisted in making one or more slots in the wing section (Fig. 1). In this way it is possible to use the wing at higher angles of attack and thus considerably increase the lift. The lift-drag ratio, however, seems to be no better in general than for ordinary wing sections. The advantage lies principally in the ability to vary the coefficient of lift, and hence the speed, within considerably wider limits. Hereby, the difficulties of taking off and landing are diminished and greater flight

---

* Reprint from "Berichte und Abhandlungen der Wissenschaftlichen Gesellschaft für Luftfahrt" (Supplement to "Zeitschrift für Flugtechnik und Motorluftschiffahrt"), No. 6, January, 1922.

speeds made possible. Our knowledge of the behavior of such slotted wings under the most diverse conditions is, unfortunately, very limited, and there is still much work to do before we shall have carried our investigations so far as to be able to choose, from the many possible modifications, the one best adapted for any given purpose.

The question of the most practical importance is what must be done in order that with an airplane we can obtain the best possible lift-drag ratio if the lift-coefficient is low and, in addition, be able to reach by easily made changes, a considerably higher coefficient of lift, where the lift-drag ratio does not need to be especially good. The former condition would be used in ordinary horizontal flight and the latter in taking off or in landing. The purely experimental solution of all the problems connected with these new wing sections is rendered very difficult by the large number of possible modifications. The most diverse cross-sections may be given the component parts of the wing and their relative size may be varied, thus bringing the slot nearer either the leading or trailing edge. Furthermore, the relative position of the parts and the width of the intervening slot may be varied. Lastly, there is the possibility of varying the number of the component wing-parts by the introduction of one or more slots. Although, for structural reasons, many forms do not come into practical consideration, the number of possibilities is still very large.

The experimental work will be considerably simplified and rendered more productive of results, if we succeed in obtaining at least an approximate idea of what takes place. We are still, however, far from being able to give a complete theoretical explanation of the phenomena of slotted wings. Nevertheless, we can contribute something toward the explanation of the unusual increase of the lift coefficient. I do not wish, however, to create the impression that what I am about to say is conclusive. I wish rather to bring the matter up for discussion, in the hope that still other viewpoints may be presented, which will help to clarify the problem.

We must first consider the question as to how it happens that, with a given wing section, the lift coefficient cannot be increased at will. In order to answer this fundamental question, we must consider more carefully the process by which lift is generated. It is known that lift is produced by the greater velocity, and consequently smaller pressure, of the air on the upper side of the wing, than on the lower (Fig. 2). This difference must vanish at the trailing edge, around which the pressures can become equalized. The difficulty lies in the fact that a strong suction must be generated on the upper side, only to vanish again at the trailing edge. From the point of least pressure on, the kinetic energy of the air must therefore be transformed into pressure by a gradual increase in the cross-section of the tubes of flow. There accordingly takes place, on the rear portion of the upper side, a phenomenon very similar to the flow through a widening tube.

Now, it is known that such a flow, in which kinetic energy is transformed into pressure, remains stable only for a very gradual increase in the size of the cross-section. If the diameter increases too rapidly, the air does not continue to flow smoothly along the wall, but separates from it and goes its own way, as a free jet, and the increased pressure is not obtained. If we increase the angle of attack of an airplane, the cross-sections of the tubes of flow on the suction-side are increased; and if a certain figure is exceeded, the air no longer flows along the upper surface of the wing, but is torn off, as it is expressed. This phenomenon is shown by Figs. 3 and 4. (The photographs were made by Dr. Heis and published in Prof. Prandtl's report on the Göttingen Aerodynamic Laboratory, in the Year Book of the Air Traffic Association, 1912-1913.) The first picture shows a wing having a normal angle of attack. The flow conforms quite well to the top of the wing and is not seriously affected by the small vortices which cover the wing. With larger wings and greater velocities, the vortices are probably still smaller. The second picture shows the same wing at a somewhat greater angle of attack, in which case the fluid no longer follows the top of the wing.

Involuntarily we now ask how it happens that the air does not separate on even a moderate increase in the diameter of the cross-section. The explanation lies in the viscosity of the air or, in most cases, more correctly, in an apparent viscosity, which, in turbulent phenomena, is conditioned by the turbulence itself. The case may be pictured qualitatively as follows: The fluid has a tendency, on account of its inertia, to flow straight ahead, instead of following the curved surface, but then there must exist, between it and the surface of the wing, a quiet or an eddying "dead-water" region. This "dead water" is now carried along by friction (or the effect of viscosity) and must be constantly replaced (Fig. 4). Now, when the viscosity is so great that, in a given time, more fluid is carried away than can flow in, the "dead water" disappears and the flow follows the surface of the wing (Fig. 3).

Such are the general outlines of the phenomena which produce lift and which also limit its magnitude. Unfortunately these phenomena cannot be treated quantitatively by theoretical methods. We must therefore content ourselves with qualitative illustrations and will now endeavor to explain, on this basis, the action of the slotted wing.

For the sake of simplicity, we will assume that there is only one slot. Such a wing section may be imagined as a biplane with a very great positive stagger and a very small distance between the wings. Some justification for this conception proceeds from the fact that, even with an ordinary biplane, the maximum lift is increased by a positive stagger. According to biplane measurements published by myself in the fourth volume of "Zeitschrift für Flugtechnik und Motorluftschiffahrt," the maximum $C_L$ without stagger was 100, with a positive stagger of 30° it was 110 and for one wing alone it was 106. Similar results were also obtained in England (Technical Report of the Advisory Committee for Aeronautics, 1915-16, Rep. 196, Sect. II). Though the differences are not great, they would evidently be greater, if the stagger were increased and the interval between the wings diminished.

We will first consider only the front wing and discuss how its characteristics are affected by the rear wing. From the theory of the biplane, we know that the flow is here obliquely upward. This affects the lift-drag ratio, but not the maximum coefficient of lift, which here alone interests us. We also know that, at this point, the flow forms a curve with the concave side up. This has about the same effect as increasing the wing camber. By increasing the latter, the maximum lift may actually be increased, though only to a very limited degree and at the expense of the lift-drag ratio. The rear wing is similarly affected by the curvature effect. It may therefore be assumed that the influence of the curvature of the flow plays a role of some importance with a given wing section with a moderately large camber, but nothing further is thereby gained than would be gained by a larger camber. The extraordinarily large increase in the maximum lift cannot therefore be thus explained.

The following consideration may be of more importance. The front wing lies in a region of increased velocity. Now, since the force of the air is proportional to the square of the velocity, it is evident that the lift on the front wing is thereby considerably increased. This argument has but one exception, namely, that the reverse is true of the rear wing, so that for the combination of the two wings the two effects neutralize each other. In calculating the relations for an unstaggered biplane, we even obtain a smaller maximum lift than for the two wings alone and this result is confirmed by experiments. The relations are, however, somewhat changed by staggering. We must go into this more thoroughly.

We will first consider the arrangement with two wings of about the same size in which the relations stand out, the clearest. The front wing, taken alone, would have a pressure distribution somewhat as shown by the fine line on the left of Fig. 5. Now, if we bring the rear wing, which has about the same pressure distribution by itself, into proximity with the front wing, the trailing edge of the latter will lie in a region of great velocity, and correspondingly small pressure, produced by the rear wing. The leading edge of the front wing, on account of its greater distance from the rear wing, lies in air that is much less disturbed and consequently in a region of nearly normal pressure. The leading edge of the front wing is, accordingly, not much affected by the pressnre on the rear wing, while the pressure on the trailing edge of the front wing is diminished. We will therefore obtain, for the front wing, a lift distribution corresponding somewhat to the dash curve in Fig. 5.

Through this modification of the pressure curve, the pressure increase on the suction (upper) side becomes much flatter. On the other hand we know that the limit of the lift is determined by the steepness of the pressure curve. It is therefore evident that we may now further increase the angle of attack, until the inclination of the pressure curve again reaches its limit value (heavy line in Fig. 5). Since the velocity has become greater everywhere, the pressure curve may climb steeper than before.

As is obvious, the lift, which is represented by the area inclosed by this curve, has become considerably greater.

Let us now turn our attention to the rear wing. Here we find corresponding phenomena. The front wing produces on the leading edge of the rear wing a decrease in velocity and a consequent decrease in the pressure diminution or suction. The trailing edge remains practically unaffected. Thus we obtain here, near the leading edge and mainly on top, an increase in pressure. The strong suction (or negative pressure) is diminished, so that here also there is a flatter pressure increase, as shown by the dash line. By increasing the angle of attack, we return approximately to the original curve, while the lift of the rear wing remains practically unchanged. Hence, in this combination, the two wings produce a greater maximum lift than when separate, the gain being principally on the front wing.

The phenomena described will perhaps be more intelligible, if we take for comparison the perfectly analogous phenomena of a simple and a compound Venturi tube. Fig. 6 shows a double Venturi tube, such as is often used on airplanes for measuring air speed. If we first imagine the small inside tube removed, we have a simple Venturi tube. The air flows through the constricted section with increased velocity and correspondingly diminished pressure. In the diverging cone behind it, the kinetic energy is again largely transformed into pressure, so that at the rear end, the external and internal pressures are again equal. Exactly the same causes which limit the lift in a wing, here make it impossible to obtain, by narrowing the throat, a pressure diminution of any desired value. In this case, however, it has long been known how to increase the suction by a suitable combination of tubes. Such an instrument is shown in the figure. The exit of the inner tube is at the point where a diminished pressure is already produced by the outer tube. The latter now forms the starting point for the further pressure diminution in the inner tube, just as in the case of the front part of the slotted wing section, which we have already considered.

We assumed in our discussion that the two parts of the wing were of about the same size. In practice, however, the front part is usually much narrower than the rear part. Our assumption that the pressures on the leading edge of the front part were not noticeably affected by the rear part, no longer holds true. Here the whole of the front section lies in a field of increased velocity and is thus able to produce a greater lift, since the lift is proportional to the square of the velocity. For the rear section, however, our previous remarks hold good. The disturbance due to the front section is felt principally on the leading edge, which therefore has approximately its normal lift. Accordingly, we even here obtain increased lift for the whole combination.

That the actual pressure distribution is approximately as described, follows from the data published by Handley-Page in "Engineering", March 4, 1921. These data are given in Fig. 7. For small angles of attack, the rear section shows about normal lift distribution. The auxiliary wing in front gives only a small lift, since its angle of attack is much too small. Only from 12° up does the lift of the auxiliary wing show any considerable increase, while the pressure distribution of the main wing remains almost the same. The increased suction on the trailing edge of the auxiliary wing is also evident. At about 20°, the flow separates from the main wing and the lift of the auxiliary wing diminishes.

The above conception of the phenomena renders the occurrence of an increased lift coefficient somewhat more comprehensible and even offers the prospect of making it possible to compute the relations. On the other hand, the following consideration may be presented. If the slot is continuously narrowed, the arguments pointing to a higher maximum lift continue to hold good, but the phenomena vanish when the slot is closed altogether. This was to to be expected from the first, since the wing is transformed by closing the slot into one of a practically normal shape. In any event, a slot of a certain minimum width is essential. Since the theory just presented, says nothing about this, the phenomena must also be considered from a different standpoint.

In explaining the phenomena of lift production, I called attention to the fact that the clinging of the air stream to, or its separation from, the upper surface of the wing depends on whether the dead air is carried off fast enough. When we consider this phenomenon on the rear section of a slotted wing, it is obvious that the work (which may be called pump-work or suction) must be performed at the expense of the kinetic energy of the thin air stream flowing through the slot. If the latter is made too narrow, the ribbon of air finally becomes so thin that its kinetic energy no longer holds out to the trailing edge of the rear section, but is, itself, transformed into dead air by mixing with the dead air above and below it. When considered from this standpoint, the phenomena of the slotted wing appear in quite another light. We can now think of this wing section as an entity, derived from an ordinary wing section by connecting its upper and lower surfaces by slots, which is, in fact, the conventional conception. The slots convey new energy to the marginal layer of air retarded by friction on top of the wing, thereby increasing its velocity and thus preventing the accumulation of dead air. The air stream flowing out of the slot acts like the jet from a syringe and reinforces the air stream on top of the wing in carrying away the dead air. Since the production of lift depends on the efficiency of this pump-work and the maximum lift is conditioned by the limited possibility of carrying off the dead air, it is apparent that any increase in the pumping efficiency increases the maximum lift.

We are now inclined to ask which of these two theories is the right one. The answer is that both are equally correct, since they both explain the same phenomena, but from different standpoints. We should rather ask which viewpoint is the more practical. To this question I would reply that we have use for both, according to what we wish to learn. The conception of the slotted wing as a biplane whose wings mutually influence each other has the advantage of enabling computation to a certain extent. With its help, we may succeed in constructing formulas which will enable the determination, in some measure, of the quantitative ralations. The second viewpoint is essential, when it is desired to form an idea of the requisite width of the slot. I would add a word of warning against too great optimism. The relations are much more complicated here, than, for example, in the theory of the monoplane or biplane. Much work must still be done, before these theories are developed into practical rules. With the limited means now available, much time will be required for this work. The immediate task is to determine whether the theories just presented really explain the essential features of the phenomena, or whether other circumstances of decisive influence will come in. This cannot be conclusively determined from the experimental data now available. If the theory, however, agrees with the facts, this is already a great gain, even though we do not succeed in working out convenient computation formulas. We then know, at least, what the essentials are for obtaining the right shapes and can thus save ourselves much useless work.

Translated by the National Advisory Committee for Aeronautics.

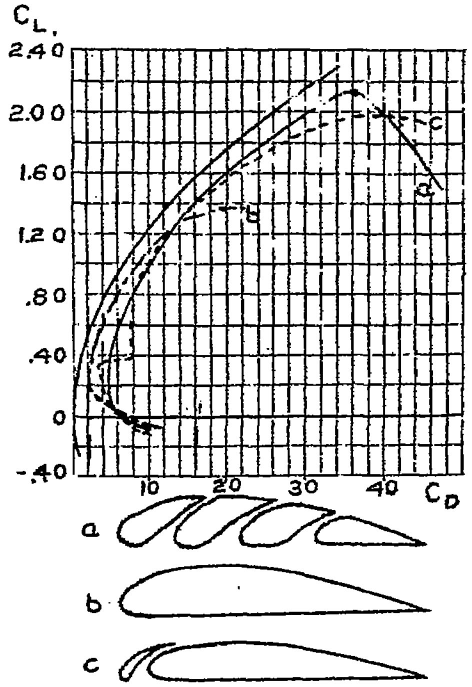

Fig. 1. Polars of an ordinary wing and of two slotted wings.

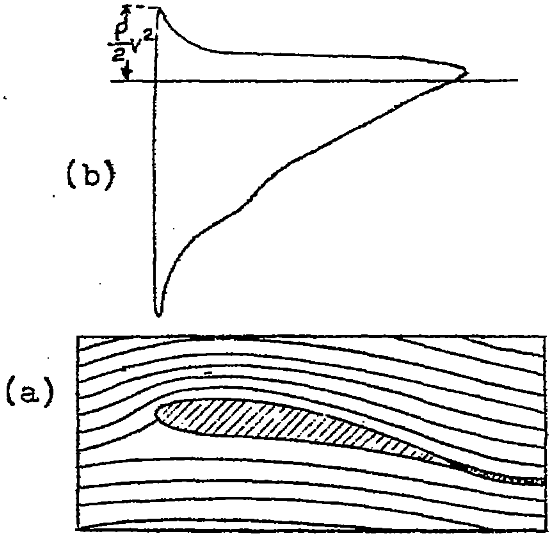

Fig. 2. (a) Flow about a wing section.
(b) Corresponding pressure distribution.

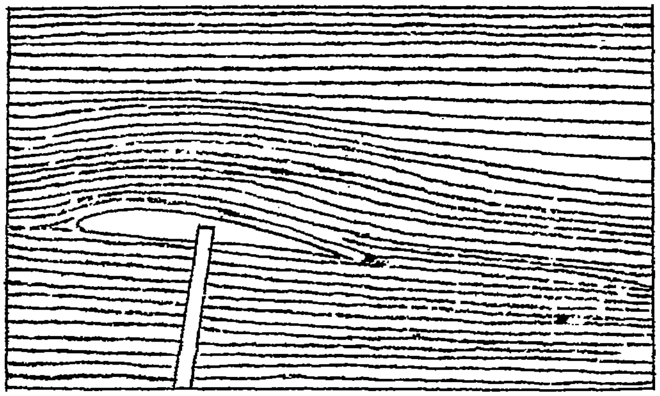

Fig. 3. Flow about a wing section at an angle of attack of 8°.

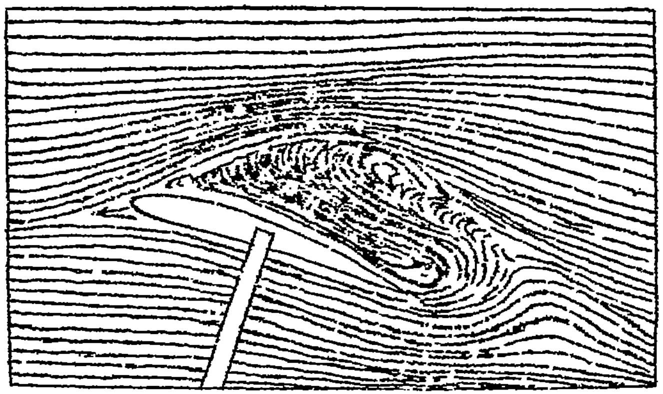

Fig. 4. Flow about a wing section at an angle of attack of 19°.

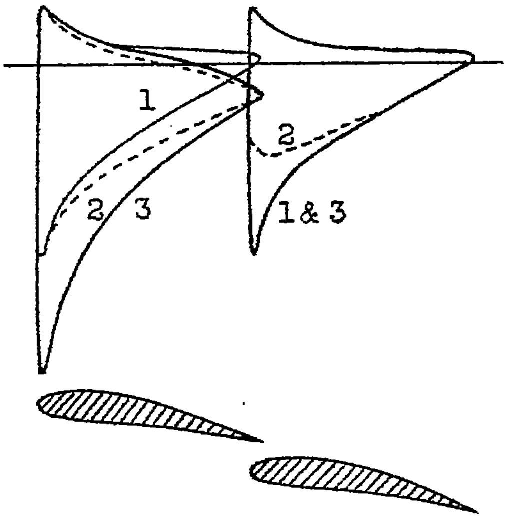

Fig. 5. Change in pressure distribution due to mutual influence of both wing sections. 1. Undisturbed pressure distribution (fine line). 2. Disturbed pressure distribution with unchanged angle of attack (dash line).

3. Pressure distribution with increased angle of attack, (heavy line). For rear wing curves 1 and 3 coincide.

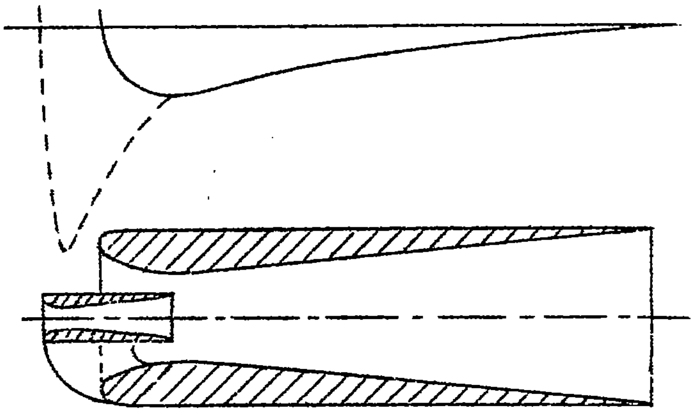

Fig. 6. Double Venturi tube with corresponding pressure distribution. (Pressure in outer tube, plain; in inner tube, dashed. )

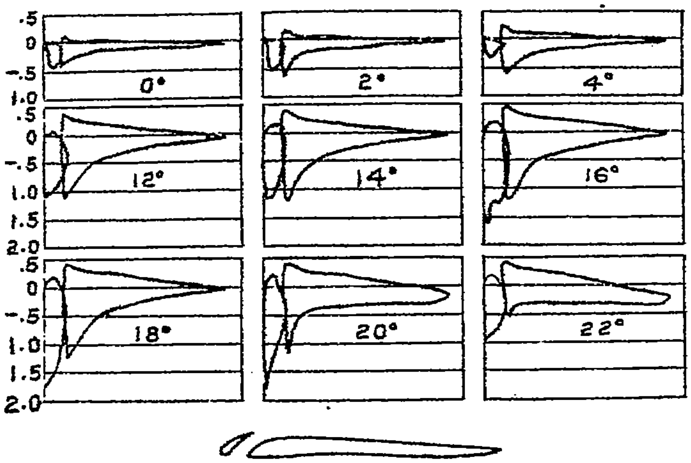

Fig. 7. Measured pressure distributions in a slotted wing.