1. Review of Aircraft Aerodynamics

For fairly obvious reasons, the wing is a major design component of any conventional and most other aircraft. Its major function, of course, is to hold up the airplane, but it also often serves as fuel container, support structure for the propulsion system, and parts of it perform control functions as well. For good reason, it is the focus of much of your major coursework. Remember, however, that the wing is not designed in a vacuum. It is only a component in a large system, and you must consider the interactions of each of the many components and subsystems in order to optimize your wing design.

1.1. Aerodynamic Lift

In Aerodynamics class, one learns that wing **lift** depends on the air density, the speed at which it is traveling, its size, its shape, and its angle of attack. A mathematical statement for the lift of the wing could be written as

$$L = \frac{1}{2}\rho v^2 S\, C_L \tag{1.1}$$

where L refers to the wing lift in pounds or Newtons, ρ is the air density in slugs/ft^3 or kg/m^3, v is the vehicle velocity magnitude (in ft/s or m/s), S is the wing area, and C_L is the wing lift coefficient. The lift coefficient depends on the actual geometry of the wing, including the airfoil shape and wing twist and taper (through the lift-curve slope, C_{L_α}), as well as the wing angle of attack, usually designated as α. It is important to remember that α is the angle that the wing zero-lift line (or, in some cases, the chord line) makes with the oncoming flow, which is not necessarily the angle with the horizontal. This would particularly be the case if the aircraft were climbing or descending, for example.

Exercise 1. *Sketch an airfoil at angle of attack, α, which is in steady, level flight at velocity, U. Then sketch the same airfoil in steady descent with forward velocity, U, descent velocity, V, and angle of attack, α.*

An expression for the lift-curve slope of a wing is given by

$$C_{L_\alpha} = \frac{\pi AR}{1 + \left[1 + \left(\frac{AR}{2\cos\Lambda}\right)^2\right]^{\frac{1}{2}}} \tag{1.2}$$

where AR is the wing aspect ratio and Λ is the wing $\frac{1}{4}$-chord sweep angle. (Define *wing aspect ratio.*)

The above discussion refers to a *finite wing*, that is, a wing of finite span. Usually, students are introduced to the aerodynamics of an infinitely long wing that can be thought of as two-dimensional since the behavior at any cross section is the same as at any other. When considering 2-D aerodynamics, an expression for the *lift per unit span* is given by

$$\ell = \frac{1}{2}\rho v^2 c_\ell c \tag{1.3}$$

where c_ℓ is the sectional lift coefficient and c is the airfoil chord. (Note that we normally use the term *airfoil* to mean a two-dimensional section and *wing* to mean the full 3-D surface.) The two-dimensional lift coefficient is expressed as

$$c_\ell = c_{\ell_\alpha}\alpha \tag{1.4}$$

Exercise 2. *What is the theoretical value of* c_{ℓ_α}*?*

The actual value of the lift-curve slope depends on the airfoil geometry. In particular, as the airfoil thickness increases up to about 0.15, the lift-curve slope also increases above the theoretical value. Beyond this thickness, however, the c_{ℓ_α} will tend to decrease because of non-ideal (viscous) effects.

At some point, equation 1.4 will no longer hold—the lift coefficient will not continue to increase as the angle of attack increases, as illustrated in figure 1.1. Again because of non-ideal effects, the airfoil will *stall* at some angle when the flow can no longer remain attached to the upper surface. The value of maximum lift coefficient is designated as $C_{L_{\max}}$ for wings and $c_{\ell_{\max}}$ for airfoils, and usually the maximum occurs at an angle just below that for stall.

1.1.1. Stall and High-Lift Devices

Within a small region adjoining the surface of a body in a flow, there exists a "boundary layer." Viscous stresses play a very important role within this region, causing viscous drag and wing stall.

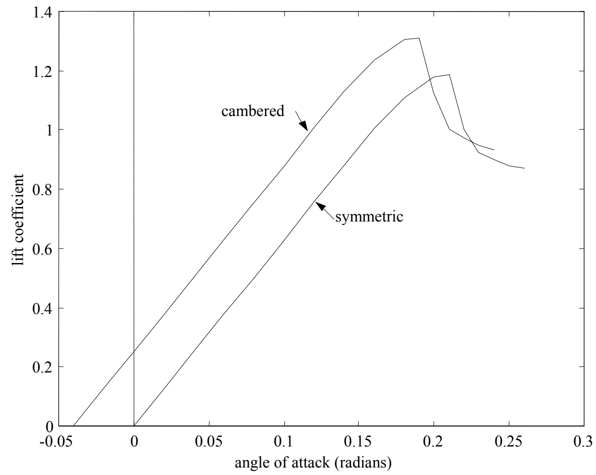

Figure 1.1: Typical lift curves for symmetric and cambered airfoils.

To illustrate the phenomenon of flow separation and stall, consider a two-dimensional flow over the top surface of an airfoil. As computed using ideal-flow theory, the pressure on the aft part of the upper surface is increasing as the flow moves toward the trailing edge. Therefore, the momentum of the flow must be enough to overcome this adverse pressure gradient. In ideal-flow theory, the flow momentum is exactly enough to return the flow to stagnation conditions at the trailing edge. In a real flow, there is an additional force acting on the air as it moves along the airfoil. This arises from the viscous stress (skin friction) at the surface, and it opposes the motion of the air and has the effect of slowing the flow velocity within the boundary layer. If the combined action of the pressure force and the friction force is enough to bring the velocity on the airfoil surface to zero, the flow will detach from the surface. This phenomenon is called "separation," and, if the separation occurs at any location not directly in the vicinity of the trailing edge, the airfoil is said to "stall." Because the flow is no longer attached to the airfoil upper surface, the ideal pressure distribution on that surface is not maintained, and lift is decreased, sometimes abruptly. Since the separated flow region is highly turbulent, stall also causes unsteady vibration and buffeting.

Exercise 3. *Sketch the C_L vs α curve for a finite wing. Do you expect the $C_{L_{\max}}$ to be higher or lower than that for the 2-D airfoil? Do you expect the angle for stall to be higher or lower than that for the 2-D airfoil?*

For most conventional and many other aircraft, the maximum available C_L plays an important role in the overall design of the aircraft. The takeoff and landing distances are directly related to the aircraft minimum speed, and a high $C_{L_{\max}}$ will allow for low-speed flight. (Use the definition of C_L to show that this is true.) The maximum lift coefficient attainable by a wing depends on the airfoil section and, to some extent, the wing geometry. However, most modern airplanes utilize high-lift devices, such as flaps and slats, to increase the $C_{L_{\max}}$ well beyond that attainable by a simple wing. The maximum C_L depends on type of flap and/or slat system, the percent chord dedicated to the flap (or leading-edge device), and the percent span that is flapped. Most aircraft do not have full-span flaps since some of the wing is required for aileron control surfaces. Some aircraft use "flaperons," a combination flap/aileron. However, the flaperon is not an effective flap relative to modern slotted flap systems.

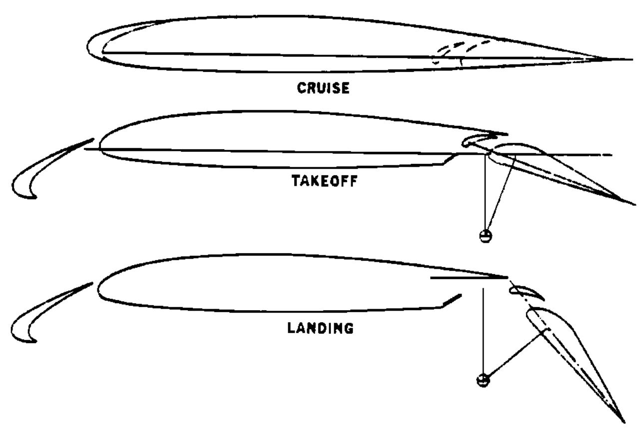

Figure 1.2: Typical double-slotted flaps/leading-edge slat high-lift system (from *Shevell,* 1979).

Flaps and slats increase the wing maximum lift coefficient in several ways:

1. Modern slats and slotted flaps extend the leading and trailing edges of the wing, creating an effectively larger wing area. Therefore, a C_L based on wing reference area will automatically be larger when the increase in area is taken into account.

2. Both slats and slotted flaps create a channel for air from the lower surface of the wing to travel to the upper surface. Especially at high angles of attack, the lower surface has much less severe pressure gradients than the upper surface; thus the boundary-layer air on the lower surface has considerably higher momentum than that of the upper surface boundary layer. This higher-momentum air is injected through these channels (or slots) into the upper-surface boundary layer, giving that air more momentumm with which to overcome the pressure gradient and remain attached.

3. Flaps add effective camber to the wing.

1.1.2. A Method for Estimating Wing Maximum Lift Coefficient

To estimate the increase in $C_{L_{\max}}$ contributed by modern high-lift devices, the following simple methodology can be used. Firstly, the change in maximum C_L is determined from

$$\Delta C_{L_{\max}} = \Delta c_{\ell_{\max}} \left(\frac{S_{\text{flap}}}{S_{\text{ref}}} \right) \cos \Lambda_{\text{hl}} \tag{1.5}$$

where $\Delta c_{\ell_{\max}}$ is the increment in airfoil maximum c_ℓ obtainable from the particular flap system, S_{flap} is the wing area spanned by the flaps, and Λ_{hl} is the sweep angle of the hinge line. $\Delta c_{\ell_{\max}}$ should be obtained from test data, but may be approximated from table 1.1. It should be noted that the table gives values for the *maximum* increase attainable at the optimum angle of attack and flap deflection. Because of the increased drag associated with such extreme flap deployment (δ_f equal to approximately 50°), this setting is normally used for landing only. Takeoff flap angles are usually in the range of 25°, with $\Delta c_{\ell_{\max}}$ equal to about 70% of the landing value.

High-Lift Device	$\Delta c_{\ell_{\max}}$
Trailing-edge flaps	
Plain	0.9
Single-slotted	1.3
Fowler (single-slotted)	$1.3c'/c$
Double-slotted	$1.6c'/c$
Triple-slotted	$1.9c'/c$
Leading-edge devices	
Fixed slot	0.2
Leading-edge droop	0.3
Kruger flap	0.3
Slat	$0.4c'/c$

Table 1.1: Increase in airfoil maximum lift coefficient for high-lift devices. (From *Raymer.*)

1.2. Aerodynamic Drag

In addition to lift, the wing and body traveling through the air generates **drag**. The drag consists of viscous or parasite drag as well as lift-induced drag. Most aircraft today travel fast enough that they also have an additional drag arising from compressibility effects.

1.2.1. Parasite Drag

Though we can investigate a lot about the forces acting on a surface by considering ideal conditions (*isentropic*), all real fluids have viscosity. Viscous stresses directly cause *skin-friction drag* and indirectly cause *pressure drag*. Skin-friction drag is simply the streamwise component of the viscous stress integrated over the surface. The viscous action creates a thin region near the surface wherein the velocity is reduced from that calculated using ideal-flow equations. This thin region is called a *boundary layer*. From the point of view of the oncoming flow, the boundary layer effectively changes the shape of the airfoil, causing thickening near the trailing edge. This new shape does not have a stagnation point at the trailing edge, so the pressure does not recover to its stagnation value there. Consequently, an integration of the streamwise component of the pressure force over the wing or airfoil does not result in zero force–hence there is a drag caused by the non-ideal pressure distribution.

Usually, it is not a simple matter to directly calculate the parasite drag. However, years and years of experiments, computation, and flight test have provided us with a very large body of data from which it is possible to make excellent predictions of parasite drag for most shapes. The parasite drag coefficient is defined as

$$C_{D_P} = \frac{D_P}{\frac{1}{2}\rho v^2 S} \tag{1.6}$$

Often, the C_{D_P} will be called C_{D_0}.

Exercise 4. *Why does the parasite drag coefficient vary with wing angle of attack?*

A more useful measure of the parasite drag is the *equivalent flat-plate drag area*, f. This quantity is exactly what it suggests—a flat plate of area, f, will have the same drag as the airplane (when the plate is positioned perpendicular to the wind). Thus, the total parasite drag is just

$$D_P = fq \tag{1.7}$$

where q is the dynamic pressure.

You can find f by doing a component drag buildup. Each exterior component of the airplane is considered separately, and the f of each is found. Then the total f is determined by summing the component drag areas. In general, the equivalent flat-plate area of the ith component can be computed from

$$f_i = C_{f_i} F_i Q_i S_{\text{wet}_i} \tag{1.8}$$

Friction Coefficient, C_f

The friction coefficient depends on Reynolds number and surface roughness, and is most affected by whether the flow is laminar or turbulent. Recall that Reynolds number is given by

$$Re = \frac{\rho v \ell}{\mu} \tag{1.9}$$

where ℓ is a characteristic length of the specific component. For wings, this would be the mean aerodynamic chord, while for fuselages or nacelles, it would be the length. The flowfield around most aircraft is largely turbulent, so the friction coefficient can be computed from

$$C_f = \frac{0.455}{(\log Re)^{2.58}} \tag{1.10}$$

This expression for friction coefficient is valid up to the transonic Mach number range.

Form Factor, F

The form factor is a measure of how "streamlined" the component is. It thus has a major influence on the pressure drag since thin bodies exhibit lower adverse pressure gradients and, therefore, less boundary-layer thickening near the trailing edge. The form factor is a function of the component thickness-to-length ratio. For wings, this value is the thickness ratio, t/c. In general, the lower the thickness ratio, the lower the form factor, though some shapes (blunt trailing edges) have higher pressure drag than others. The following give form factors for the major aircraft components:

$$F_{\text{wing}} = 1 + Z\frac{t}{c} + 100\left(\frac{t}{c}\right)^4 \tag{1.11}$$

where

$$Z = \frac{(2 - M_\infty^2)\cos\Lambda}{\sqrt{1 - M_\infty^2 \cos^2\Lambda}} \tag{1.12}$$

This equation can also be used to find the form factors for tail surfaces, pylons and struts.

$$F_{\text{fuselage}} = 1 + \frac{60}{(\ell/d)^3} + \frac{\ell/d}{400} \tag{1.13}$$

where ℓ/d is the fineness ratio:

$$\ell/d = \frac{\ell}{\sqrt{(4/\pi)A_{\text{max}}}} \tag{1.14}$$

A_{max} is the maximum cross-sectional area. Equation 1.13 is also appropriate for smooth canopies and pods. (For transport-type canopies, and additional 0.07 times the canopy area should be added to the f_{canopy}.) The form factor should be increased by about 40% for square-sided fuselages and by about 50% for flying boat hulls. This expression is appropriate for external stores such as auxiliary fuel tanks. Equation 1.13 can also be used for nacelles, but the effective fineness ratio should be computed from

$$\ell/d_{\text{nacelle}} = \frac{\text{nacelle length} + \text{leading-edge diameter}}{\left\{\frac{4}{\pi}\left[A_{\max} - \frac{A_{\text{exit}} + \frac{A_0}{A_{\text{LE}}} A_{\text{LE}}}{2}\right]\right\}^{\frac{1}{2}}} \tag{1.15}$$

where A_{exit} is the exit area, A_0/A_{LE} is the inlet mass flow ratio (≈ 0.8) and A_{LE} is the area at the leading edge ($\approx 0.7A_{\max}$).

Interference Factor, Q

The airplane components don't just exist by themselves. Drag depends on not only the component size and shape, but also on aerodynamic interference between the component and its surrounding components. For example, the dynamic pressure can be increased or reduced at a junction between a fuselage and tail surface, which alters the drag of the tail relative to its isolated drag. Interference factors tend to have values ranging from about 1 for the fuselage and well-filleted wing to about 1.5 for fuselage-mounted nacelles.

Wetted Area, S_{wet}

The air can only create stress on surfaces that it touches, so the relevant area over which the friction or the pressure will act is the *wetted area*—the actual area exposed to the air. This value is completely determined by the geometry of the aircraft and, in actuality, is quite difficult to calculate. CAD packages can make the process easier.

Miscellaneous Parasite Drag

The drag of some items does not quite fit the form of eq. 1.8 and so must be computed in an alternative manner. Table 1.2 gives values of f/A_{frontal}, the equivalent flat plate drag area normalized by the projected frontal area, of landing gear components. Deployed flaps have a drag area given roughly by

$$f_{\text{flaps}} = 0.0023\frac{b_{\text{flap}}}{b}S_{\text{ref}}\delta_{\text{flap}} \tag{1.16}$$

with b_{flap} representing the span of the flap and δ_{flap} the flap angle in degrees. Recall that takeoff flap setting is approximately 25° whereas a $\delta_{\text{flap}}$of 50° is used on landing. Fuselage-mounted speed brakes have $f/A_{\text{frontal}} = 1$, while wing-mounted speed brakes, or spoilers, have $f/A_{\text{frontal}} = 1.6$.

Component	f/A_{frontal}
Regular wheel and tire	0.25
Second wheel and tire in tandem	0.15
Streamlined wheel and tire	0.18
Wheel and tire with fairing	0.13
Streamlined strut	0.05
Round strut or wire	0.3
Flat-spring gear leg	1.4
Fork, bogey, irregular fitting	1.0–1.4

Table 1.2: Estimated equivalent flat-plate drag areas for landing gear components. (From *Raymer*.)

Base area, A_{base}, refers to aft-facing flat surfaces. Any such surface adds pressure drag to the vehicle since the lack of streamlining causes flow separation and low pressure acting over the area. Any component with an aft-facing angle of more than 20° should be considered to have separated flow. In such a case, the base area is defined as the projected area of the surface having greater than 20° slope. Base drag can be computed from

$$f_{\text{base}} = \left[0.139 + 0.419(M_\infty - 0.161)^2\right] A_{\text{base}} \tag{1.17}$$

If the fuselage has upsweep, the additional aft-surface drag should be determined using

$$f_{\text{ups}} = 3.83u^{2.5}A_{\text{max}} \tag{1.18}$$

where, now, $A_{\text{max}}$is the maximum fuselage cross-sectional area.

Windmilling Engine and Propeller

When computing engine-out performance, it is necessary to include additional drag from stopped or windmilling engines or propellers. While operating, these propulsion components are considered to have no drag since the reported thrust includes a decrement for forces generated in the drag direction. Propeller drag depends on the solidity, σ, given by

$$\sigma = \frac{Bc_{\text{avg}}}{\pi R} \tag{1.19}$$

where B is the number of blades, c_{avg} is the average blade chord, and R is the propeller radius. Note that this quantity just equals the ratio of blade area to disk area. The drag area is given by

$$f_{\text{prop}} = \begin{cases} 0.1\sigma A_{\text{disk}} & \text{feathered} \\ 0.8\sigma A_{\text{disk}} & \text{stopped} \end{cases} \tag{1.20}$$

A windmilling jet has an equivalent drag area given by 0.3 times the face area.

Leakages and Protuberances

Suring the initial stages of design, it is impossible to account formally for all contributors to the parasite drag, particularly that caused by antennas, probes, windshield wipers, and miscellaneous inlets and exits. Additional drag from these items can be from 2–10% of the total parasite drag. For new designs, it is recommended to add 2–5% of the total for jet transport aircraft and 5–10% for propeller aircraft and jet fighters.

1.2.2. Induced Drag

It is shown in aerodynamics class that an infinitely long wing can be modelled as an infinitely long vortex (or infinitely long vortex sheet stretching between the airfoil leading edge and the trailing edge). Helmholtz tells us that a vortex cannot end in space, so we cannot just cut off the vortex (or sheet) at the wing tips when we want to analyze finite wings. Thus, the vortices must go somewhere, and the only logical place for them to go is to trail behind the wing to infinity. (Some other theorems about vortices justify this conclusion. It also makes sense.) Biot and Savart tell us that vortices induce velocities—the vortices trailing behind the wing create an additional velocity field. The magnitude of the induced field tends to be small, but its direction at the wing is downward. The net effect of this induced velocity is to effectively change the direction of the oncoming flow relative to the wing. Thus, the wing experiences a flow at a lower angle than it would without the induced effects, reducing the lift by a small amount and, more importantly, adding a drag component to the aerodynamic force. (Recall that $\mathbf{f} = \rho \mathbf{v} \times \mathbf{\Gamma}$, so that the aerodynamic force is generated perpendicular to both the circulation vector *and* the effective velocity.)

This drag is called *induced drag.* For a given lift and a given wing span, it is at a minimum when the induced velocity is a constant function of span. In this ideal situation,

$$D_I = \frac{L^2}{q\pi b^2} \tag{1.21}$$

If the lift is not distributed ideally (meaning the induced velocity is not constant) the drag increases. Though it is not too difficult to compute the induced drag for different wing configurations, the usual practice for purposes of conceptual design is to use the "Oswald efficiency factor:"

$$D_I = \frac{L^2}{q\pi b^2 e} \tag{1.22}$$

For standard aircraft, e is usually a number between 0.7 and 0.85, and it depends on wing twist, taper, aspect ratio, and the fuselage diameter to wing span ratio. Methods for estimating e can be

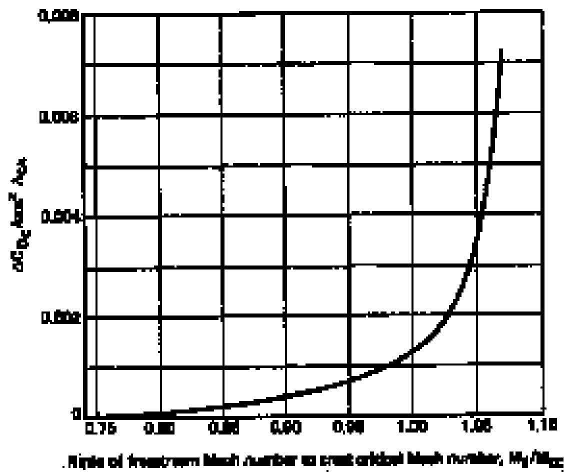

Figure 1.3: Additional compressibility drag as a function of fraction above critical Mach number and wing sweep (from *Shevell,* 1983).

found in many design textbooks.

Exercise 5. *Develop the expression for induced drag* ***coefficient****. Explain why for a given airplane weight, the (ideal) induced drag does not depend on the aspect ratio.*

1.2.3. Compressibility Drag

The term "compressibility drag" is somewhat of a catchall that accounts for the increase in drag above and beyond that encountered at flight Mach numbers of 0.3 and below. (Define *Mach number.*) In fact, the rise in drag coefficient is not noticeable until the aircraft reaches its "critical Mach number," or the flight Mach number at which sonic flow first occurs somewhere on the aircraft. The appearance of sonic flow is important because it signals the formation of shock waves wherein the flow suddenly slows to subsonic speed through a very short distance. This process is not reversible and not isentropic. Consequently, flow energy is lost as heat and the fluid behind the shock wave cannot recover the initial pressure as it reaches the rear stagnation point. (You can think of the mechanical energy per unit mass as being potential (p) plus kinetic ($\frac{1}{2}\rho v^2$). Once some of this mechanical energy is lost to heat (irreversibly), it is clear that the pressure at a stagnation point ($v = 0$) behind the shock will be lower than that in front of the shock. Hence, there will be a net force acting on the body in the upstream direction.)

Since the mechanism causing compressibility drag is non-ideal, it is not simple to compute (though it is quite possible, especially for not-too-complicated geometries). Again, years of testing and experimentation have provided us with accurate methodologies for predicting the compressibility drag for transport and military aircraft. The value of the ΔC_{D_c} depends on how far above critical Mach number, M_c, the aircraft is flying. (Aircraft are usually designed so that the wing reaches M_c before other components.) M_c depends strongly on the wing thickness and sweep, with low thickness and high sweep increasing the value of the critical Mach number.

Exercise 6. *Explain why the ΔC_{D_c} increases with increasing C_L.*

1.2.4. Aircraft Total Drag

In standard form, the aircraft total drag coefficient is written as

$$C_D = C_{D_P} + \frac{C_L^2}{\pi AR\, e} + \Delta C_{D_c} \tag{1.23}$$

This equation is often called a *drag polar*. The plot of C_D vs C_L results in approximately a parabola, though the dependence of C_{D_P} and ΔC_{D_c} on lift coefficient causes deviation from the purely second-order polynomial relationship. It is often more convenient to express the drag polar as

$$C_D = C_{D_0} + \frac{C_L^2}{\pi AR\, e'} + \Delta C_{D_c} \tag{1.24}$$

where C_{D_0} is the *zero-lift* drag coefficient and e' accounts for the lift dependence of the parasite drag as well as the non-ideal lift distribution. A somewhat simplified expression for e' can be calculated from

$$e' = \frac{1}{\pi AR \left\{ \frac{1}{\pi ARus} + .38 C_{D_0} \right\}} \tag{1.25}$$

In this equation, u represents the non-ideal effects associated with wing planform (taper, AR, sweep), and s accounts for the effect of the fuselage. If the wing is swept 20° or less and the taper ratio is 0.25 to 0.5, an initial estimate of $u = 0.99$ is valid. For higher values of sweep and lower or higher values of taper, this value should be estimated using a finite-wing calculation or prepared charts. s depends only on the fuselage diameter to wing span ratio, $\overline{d}$:

$$s = 1.6561\overline{d}^3 - 2.5407\overline{d}^2 + 0.0393\overline{d} + 1 \tag{1.26}$$

Strictly speaking, this formula should only be used for $\overline{d} \leq 0.3$.

1.3. Aerodynamic Moment

Usually the aerodynamic moment of a wing or airfoil is taken about the leading edge or the $\frac{1}{4}$-chord. The distribution of pressure on an airfoil may generate a moment even if the lift is zero. The moment is important in conducting stability and trim analyses of an aircraft. Cambered airfoils have nose-down pitching moment (at zero lift), while symmetric airfoils have zero moment. Trailing-edge flaps effectively add camber, and can generate extremely large nose-down moments. A conventional aircraft can trim in the presence of such moments by using the horizontal tail to balance the aircraft about the center of gravity.

Terminology associated with moment generation includes *center of pressure* (the location at which the lift effectively acts, meaning the moment about that point is zero, by definition) and *aerodynamic center* (the point about which the moment does not change with angle of attack). Moment is made dimensionless as follows:

$$C_M = \frac{M}{\frac{1}{2}\rho v^2 S\, c} \tag{1.27}$$

It is usually necessary to specify the point about which moments are taken.

Exercise 7. *Show how to obtain the location of the center of pressure for an airfoil.*

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