

EXPERIMENTS ON THE ORIGIN OF TIP-VORTICES

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ABSTRACT[†]

Measurements of the structure and evolution of the tip vortex from a rotor in low-speed forward flight are presented. On the advancing blade spanwise flow on the pressure side of the blade is comparable in magnitude to the inflow. Core circumferential velocity profiles show multiple features during formation. Trajectories and peak core velocities of the tip-vortex during its formation on top of the blade are presented. The peak core circumferential velocity is constant during vortex formation. Peak core axial velocity tends towards tip-speed near the blade tip. In the near wake, the variation in core axial velocity exhibits a periodic behavior. Circulation measurements from different rotary-wing tests are correlated with an empirical model that was developed to relate the tip-vortex circulation to the blade parameters. The trailed circulation is seen to be only about 40% of the bound circulation computed using the standard assumptions for calculation of blade loads.

INTRODUCTION

Though studied for well over a century, the origin, structure and evolution of the tip vortex from a lifting surface remain poorly understood. Vortex characteristics are of first-order importance in many problems of aerodynamics and aeroacoustics. Examples are blade-vortex interactions in rotorcraft (Yu [1], Caradonna et al.[2]) and trailing-aircraft vortex hazards which limit air traffic at airports, Rossow[3], Hinton[4]), in fixed-wing aircraft. Much of the current understanding of vortex generation and core structure comes from experiments on fixed wings. However, the structure of the near wake depends on the blade loading distribution, where there are substantial differences between fixed and rotary wings (Figure 1). Thus, modeling rotary wing vortices on the basis of fixed wing data could produce erroneous conclusions.

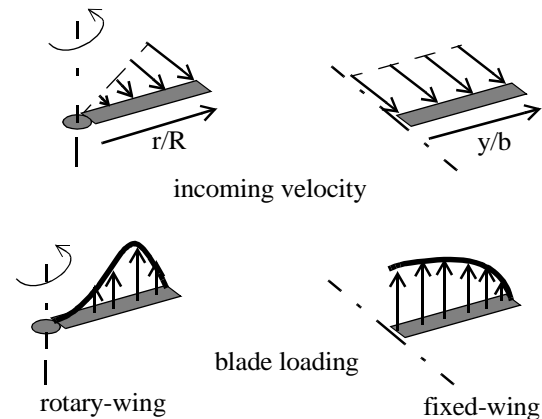


Figure 1: Spanwise loading distribution in a fixed wing compared to a rotary wing

One issue often ignored in rotary-wing vortex modeling is the presence of a substantial velocity component directed along the axes of the strong vortices in the rotor wake (Figure 2). Core axial velocity is of first-order importance to vortex interaction problems (Mahalingam et al.[5]). The lack of implementation of core axial flow in vortex models is attributable to the widely varying results obtained in studies of core axial velocity. Theoretical work by Batchelor[6] and Saffman[7] postulated a jet-like or wake-like flow in the core of a wing tip-vortex depending on a parameter which was a function of the tip loading. Experiments on fixed wings have shown both jet-like and wake-like core axial velocities([8]-[15]). There have been several hypotheses for the reasons behind the observed effects. Some researchers have reported an increase in wake deficit with Reynolds number. Others have observed dependence on the tip-shape and airfoil section used. The loading distribution at the tip is sensitive to the tip-shape. There have been studies where the measurements showed core axial flow reversing direction during its downstream development. All the results indicate that the axial velocity in the tip-vortex core depends on the nature of the roll-up process

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at the blade tip. Thus it is necessary to understand the vortex formation process to clearly understand the causes of core axial flow, and enable prediction of vortex characteristics for future blade designs.

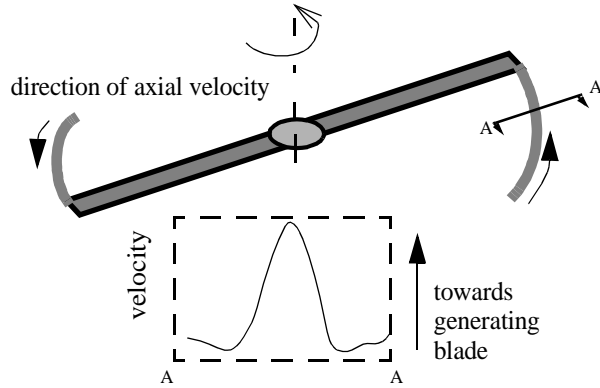


Figure 2: Core axial flow in a rotary wing tip-vortex

Rotary Wing Data

Measurements by McAlister et al.[16] on a two-bladed rotor in hover, showed strong wake-like axial velocity. Shivananda[17] used a split-film anemometer to resolve all three components of velocity in the wake of a single-bladed, square-edged rotor in hover. He found a wake-like core. Thompson[18] studied the same rotor blade wake, and resolved all three components of velocity in the vortex using a laser velocimeter. He showed not only a wake-like vortex core, but also secondary features inside the core, indicating several layers of vortex sheet roll-up. Flow visualization evidence in the literature on propeller wakes (Adams and Gilmore[19]) supports this finding. Leishman et al.[20] measured the 3 components of velocity in the tip vortex core of a 1-bladed rotor in hover and found a wake-like axial velocity. The axial velocity profiles dissipated within the first 90 degrees of wake age. The core diameter increased while the core peak tangential velocity decreased rapidly in the first 180 degrees of wake age.

For ease of measurement, some researchers have used oscillating wings to represent the time-varying nature of the tip-vortex from a rotor in forward flight. Recent measurements on oscillating wings have shown wake-like core axial velocities. Measurements by Ramaprian and Zheng[21] indicate a core axial

velocity deficit increasing with the pitch angle. Similar measurements by Chang and Seung[22] show core axial velocity deficits approaching the freestream velocity. There are large variations in the core properties measured in different tests over a range of conditions.

As part of an effort to understand the nature of vortex formation a database for rotary, fixed and oscillating-wing vortices has been setup in order to integrate observations from various tests over decades. This knowledge-base serves as a look-up table for vortex-data, and helps compare data from different configurations. This web-site can be accessed at <http://www.ae.gatech.edu/research/windtunnel/vortorgn/vortorgn.html>

The link Vortex Data From Several Tests links to tabulated data for fixed, oscillating and rotary wings. There are a large number of tests on tip-vortices over a wide range of test-parameters. This is shown in Figures 3 and 4 for three parameters; aspect ratio, Reynolds number, and thrust coefficient. Aspect ratios tested vary between 5 for model rotors to 20 for full-scale rotors. Tests have been conducted over an order of magnitude variation in Reynolds number. Thrust coefficients vary over an order of magnitude. Despite this large range of test data, no generalized theory for vortex formation and evolution has been developed. In the next section, an empirical model is developed for correlating the tip-vortex circulation with the blade loading parameters, in order to bring together the large parametric variations noted above.

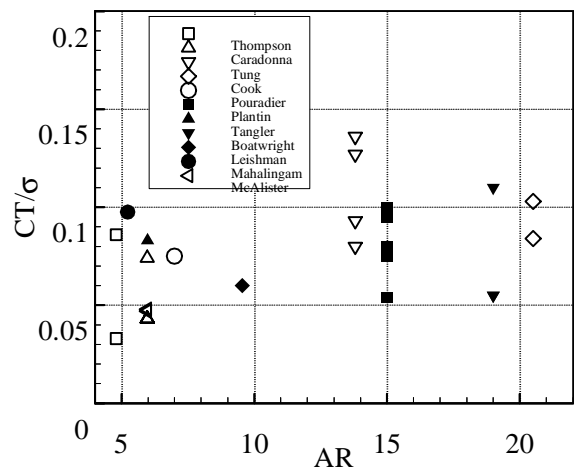


Figure 3: Data range of aspect ratio and thrust coefficients

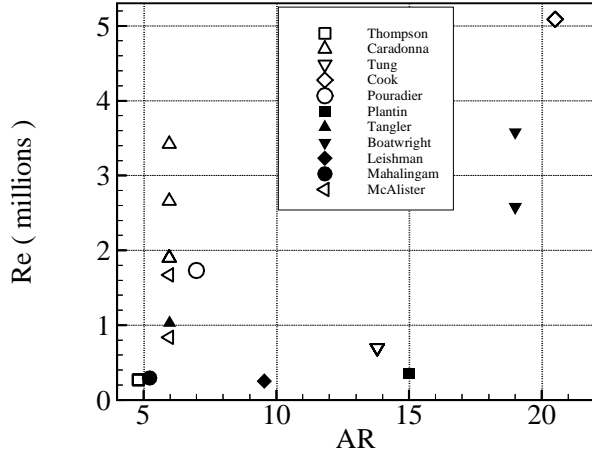


Figure 4: Data range of aspect ratio and Reynolds numbers.

MODEL FOR ESTIMATION OF TIP-VORTEX STRENGTH

Most vortex models that are used today are based on the Betz roll-up model or a variation thereof, resulting in a conclusion that the tip-vortex strength is equal to the peak bound circulation. This section presents an expression to estimate the circulation contained in the tip-vortex in terms of the Reynolds number, aspect ratio and the geometric angle of attack of the generating wing. Circulation data from several researchers is compared to this theoretical model in the results section. Four primary factors are identified as having first order effects on the circulation of a wing tip vortex: the aspect ratio, Reynolds number, downstream distance and thrust coefficient. In this derivation, downstream distance will be excluded.

Let Γ_V be the strength of the tip-vortex after roll-up is complete and let Γ_{max} be the maximum bound circulation on the rotor blade. Then, as a general case,

$$\Gamma_V = k \Gamma_{max}$$

Let the spanwise location on the rotor blade where the maximum bound circulation occurs be R^* . Then, at $r = R^*$, the Kutta-Joukowski theorem gives,

$$\rho U^* \Gamma_{max} = \frac{1}{2} \rho U^{*2} c^* C_l^*$$

where c^* is the chord length, U^* is the

incoming velocity, C_l^* is the lift coefficient of the blade section at $r=R^*$. This results in an expression for the trailed vorticity in the vortex,

$$\Gamma_V = \frac{k}{2} (U^* c^*) (a^* \alpha^*)$$

where, a^* is the section lift curve slope, α^* is the section angle of attack. Letting $Re^* v = U^* c^*$, and absorbing $a^* k = k_1$,

$$\Gamma_V = \left(\frac{k_1}{2} \right) (Re^* v) (\alpha_g - \alpha_i)^*$$

where α_g is the geometric and α_i is the induced angle of attack. Now, if we make an assumption that the induced angle of attack is primarily due to the tip vortex and is of the form,

$$\alpha_i^* = \frac{V_i}{U^*}$$

and we assume most of the induced velocity is caused by the tip-vortex and use the span b as a characteristic length,

$$V_i = \frac{\Gamma_V}{k_2 b}$$

$$\alpha_i^* = \frac{\Gamma_V}{k_2} \left(\frac{1}{AR Re^* v} \right)$$

Substituting this into the expression for the tip vortex strength, we get,

$$\Gamma_V = \left(\frac{k_1}{2} \right) (Re^* v) \left(\alpha_g - \frac{\Gamma_V}{k_2} \left(\frac{1}{AR Re^* v} \right) \right)^*$$

Simplifying this for the tip-vortex strength Γ_V , and absorbing all the constants into two constants, K and $K1$, we get,

$$\Gamma_V = (K) (Re^* v) (\alpha_g)^* \frac{1}{\left(1 + \frac{K1}{AR} \right)}$$

This expression indicates that the tip vortex circulation should be proportional to $Re^* v$ based on chord, the geometric angle of attack and AR. The utility of this result is that the tip-vortex strength is related directly to the blade loading parameters at one rotor

blade section. $Re \cdot v$ and section angle of attack can be easily replaced by corresponding values at the tip and the factors can be absorbed into the constants. Note that since Reynolds number occurs as a product with kinematic viscosity, viscous effects do not play a role in determining the circulation in the tip-vortex. Circulation non-dimensionalized by tip-speed and chord is independent of Reynolds number and depends only on section angle of attack. Thus the Reynolds number is only used to reduce the number of independent parameters by one.

EXPERIMENTS

Until flow computations become sufficiently reliable, there is a need for experimental data to understand vortex formation and evolution. Three parameters are critical for modeling the vortex wake accurately; 1) the location (x,y,z coordinates) where the vortex first forms, 2) its strength, and the variation thereof, in the initial portion of the near wake, and 3) its trajectory in the near wake. In addition, to compute blade loads due to close passage of vortices, it is also necessary to know more about the interior structure of the vortex.

Laser velocimetry was performed near the blade tip on the advancing side of a two-bladed isolated rotor in forward flight. The configuration is described in Figure 5. The test conditions and model parameters are given in Table 1.

Table 1: Test conditions and model parameters

Freestream velocity	5 m/s +/- 0.25%
Rotor RPM	1050 +/- 1
Rotor collective pitch	10 degrees
Rotor diameter	0.9144 m
Tip path inclination	4.59 +/- 0.1 degrees
Airfoil section	NACA0015
Tip chord	0.0857 m
Tip-speed	50.27 m/s
Reynolds number based on tip chord	2.87×10^5

The rotor blades are untapered, untwisted, with a constant chord. The rotor has a solidity of 0.12. The rotor shaft is inclined at 6 degrees to simulate forward flight. Rotor rotation is counter-clockwise viewed from above. The rotor system is instrumented with two accelerometers for vibration monitoring and balancing, and an optical trigger that produces a TTL pulse each time the rotor moves through zero degrees of rotor azimuth. The trigger is used for phase averaging during data-acquisition.

The measurements were performed on the advancing blade side (ABS) of the rotor, $\Psi = 90^\circ$, with a fiber optic probe placed outside the wake on the ABS and mounted on a traverse with three degrees of linear motion. The vertical and streamwise components were measured by mounting the probe horizontally. The lateral component was measured by mounting the probe vertically and directing the laser beams from under the rotor, while staying away from the wake-boundaries. It was ensured that the same origin was used for each of the components. Since turbulence measurements were not of interest in these tests, it was considered sufficient to measure the velocities component by component and combine to get the 3-D velocity field.

Measurements of vortex characteristics are often severely affected by factors such as facility unsteadiness, seed particle dynamics and probe-vortex interference. In many test facilities, the flowfield of a rotor does not repeat with the expected "n-per-rev" periodicity. The usual reason for this is that the strong vortices in the wake interact with the facility walls, and cause long-time-scale fluctuations which appear to be chaotic. When single-point measurements are conducted in such flowfields, and phase-resolved ensemble-averaging is used to obtain velocity fields, the vortex features appear to "diffuse" or dissipate rapidly, since the high gradients are smeared over an uncertain number of phase intervals. The flow-field in forward flight in this tunnel has been shown to be periodic to better than 1 degree of rotor azimuth[26] and thus single component measurements are justified.

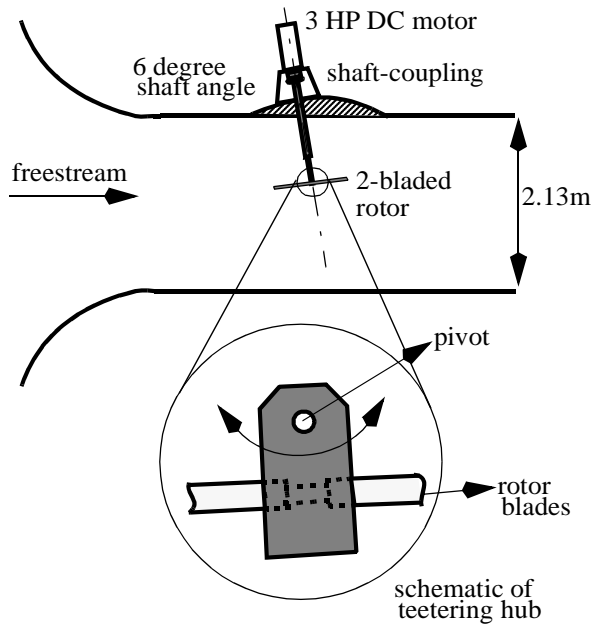


Figure 5: Isolated rotor setup in the Harper Wind-Tunnel

RESULTS AND DISCUSSION

Velocity measurements on the advancing blade side of the rotor

Figures 6a-b show the development of the velocity field in a spanwise-vertical plane at two chordwise locations on the rotor blade tip as it passes through the advancing blade side (ABS). The measurement plane was aligned parallel to the trailing edge of the blade at the ABS. Figure 6a is at $x/c=0.628$ and Figure 6b is at $x/c=1.0$. Three dominant characteristics of the velocity field are shown. The first is the strong inflow, caused by the blade passage. The second is the roll-up occurring at the blade tip. The third is the large spanwise velocity on the pressure side of the rotor blade. Note that the spanwise velocities are large only during the time the blade passes through the measurement region. This suggests that the flow on the rotor blade is highly three-dimensional and 2-D blade element methods might not be sufficient to calculate blade loads. The spanwise velocities are presumably set up by centrifugal effects. On the pressure side the flow due to the centrifugal effects and that set up by the pressure difference across the upper and lower surfaces are in the same direction, resulting in strong spanwise flow. On the suction side, the velocities due to roll-up are opposite in direction to the centrifugal effects, resulting in weak spanwise flow. The vortex is formed on the suction side of the rotor-blade inboard of the tip.

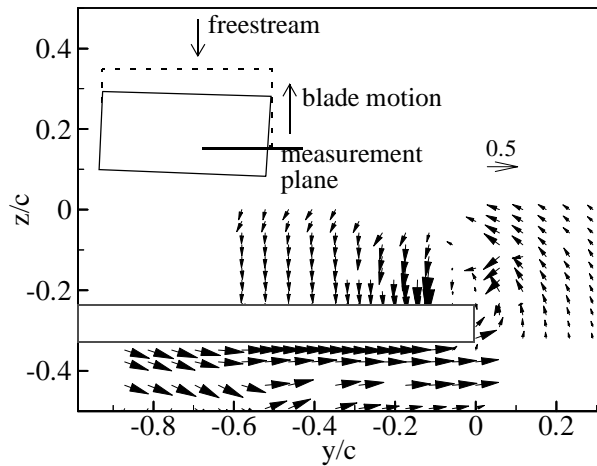


Figure 6a : Circulatory velocity development in the tip vortex over the blade tip, $x/c = 0.628$

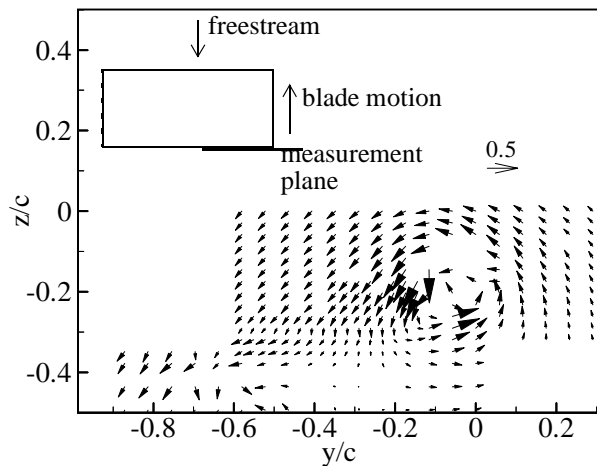


Figure 6b : Circulatory velocity development in the tip vortex over the blade tip, $x/c = 1.0$

Figure 7 shows the development of the circulatory velocities in the vortex core. The velocity profiles become smoother as the trailing edge of the rotor blade is reached. Even at $x/c = 0.628$, several secondary features are observed. These could be related to the rolling up of discrete sheets of the shear layer at the blunt tip of the blade, but need to be studied further using measurements with higher spatial resolution. Also shown in the figure is the variation of the spanwise component of velocity along the pressure side of the rotor blade. As noted earlier, the spanwise component is comparable in magnitude to the inflow and cannot be neglected in the calculation of blade loads.

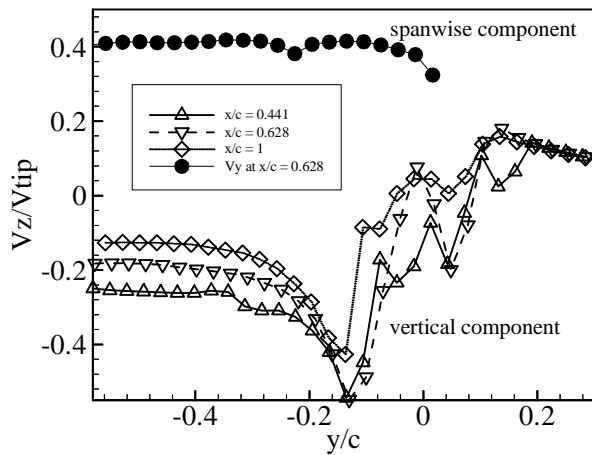


Figure 7: Velocity profiles during vortex formation

Figure 8 shows the trajectory of the tip-vortex during the developmental stages over the tip of the rotor blade. The vortex center moves inboard by 1% of blade radius (5% of blade chord) and lifts up from the surface by 1.6% of the blade radius (8% of blade chord) by $x/c = 0.8$. In the first 0.2 chord lengths behind the trailing edge there is no significant radial motion, but there is a small downward motion. Figure 9 shows the magnitudes of peak core axial and circumferential velocity developing over the blade tip. The peak core axial velocity tends to tip-speed over the blade tip, before becoming level at 0.2 V_{tip} . This is interesting, since the core axial velocity was seen to peak up to 50% of the tip speed in the near-wake in the same rotor configuration. Core circumferential velocities reach a value of about 0.4 V_{tip} which stays constant all the way to the trailing edge.

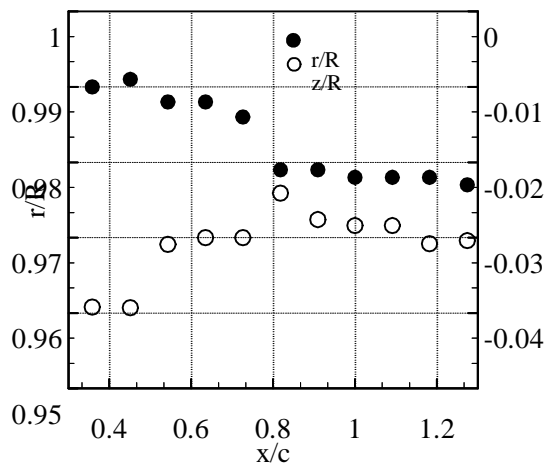


Figure 8: Trajectory of the tip-vortex during vortex formation

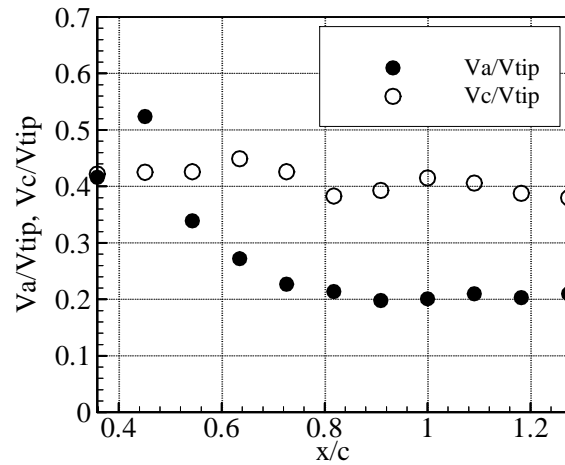


Figure 9: Development of core axial and circumferential velocity during vortex formation

Development of core axial velocity in the near wake

Figure 10 shows the variation in the axial velocity in the tip-vortex at three azimuths of the rotor cycle, ABS, RBS and rear of the rotor. The vortex from the same blade has been chosen for comparison at all the azimuths. In all the cases, there is a distinct periodic pattern to the core axial velocity evolution. The period of this pattern is about 10 degrees of rotor azimuth, or a time of 1.62ms. The Strouhal No. based on this period is 0.16, which suggests a shedding pattern very similar to the shedding in the wake behind a cylinder. Periodic shedding of the boundary layer fluid near the blade tip followed by roll-up of filaments of vorticity of same sign would result in discrete vorticity rolled around the tip-vortex, resulting in a periodic pattern to the core axial velocity. A schematic of the formation process is shown in 3 stages in Figure 11. In stage one, the 3-D boundary layer on the rotor blade rolls over the tip. In stage two, the rolled up boundary layer detaches from the blade, resulting in a spiral vortex sheet wrapped around the vortex. In stage three, this sheet of vorticity rolls up into a discrete ring vortex in a fashion reminiscent of the roll-up of a free shear layer. This suggests a vortex model as shown in Figure 12 which has been implemented by Radcliff et al.[35]. Thus, there appear to be two primary factors that determine the core axial flow. At the initial vortex ages, the no-slip condition is responsible for high core axial velocities. Within the first 6 degrees, however, the peak axial velocities reduce substantially and their evolution has a wave-like pattern, strongly suggesting the presence of discrete filaments rolled around the tip-vortex.

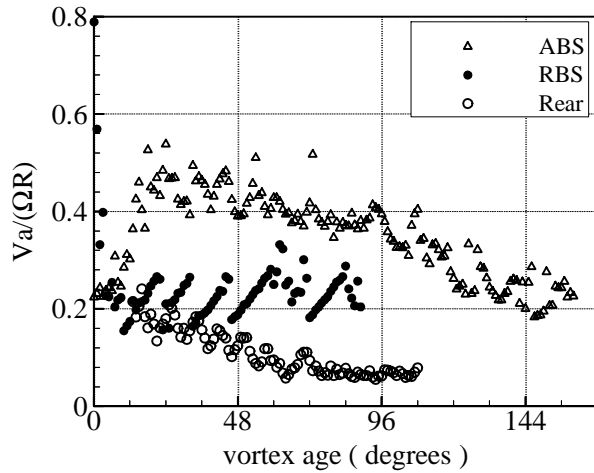


Figure 10: Variation in core axial velocity in the nearwake

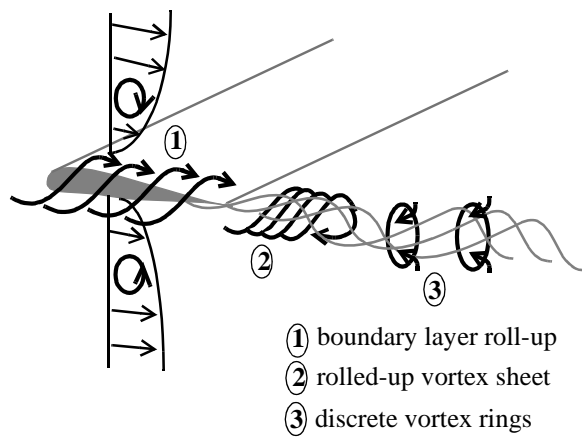


Figure 11: Proposed theory for generation of core axial velocity

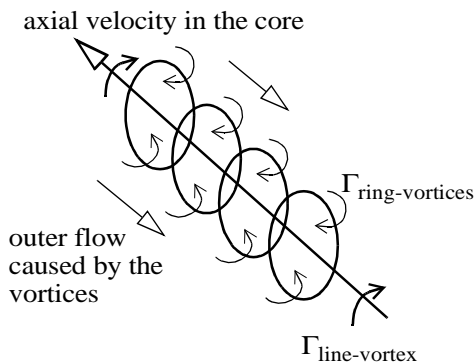


Figure 12: Vortex model developed by Radcliff et al.[35]

Correlation of circulation measurements from several rotary wing tests

The relation developed in the previous section was used to obtain the theoretical circulation for several tests on rotary wings by substituting the experimental test conditions in the expression for vortex circulation. Figure 13 shows the plot for the theoretical circulation versus the measured values. The experimentally measured circulation values match closely with the theoretical circulation. Significantly, it was found that data belonging to a particular test clumped together on one side of the theoretical line. The deviation from the straight line is a result of parameters not included in the derivation, including the age of the vortex when it was measured, as well as facility unsteadiness and measurement uncertainties. The measured data available fits the above expression very well for values of $K = 1.24$ and $K1 = 1$. Using this value of K , we get $k = 0.395$. This implies that the circulation in the tip-vortex is only about 40% of the maximum bound circulation on the rotor-blade!

The reasons for this can be partly attributed to the approximations in the calculation of the bound circulation distribution. The flow field in which the rotor blade operates is highly unsteady, with a time varying freestream as well as inflow. Added to this is the complexity of the three-dimensional velocity field that exists over most of the rotor blade. The use of the Kutta-Joukowski theorem coupled with blade element theory is an oversimplification of the flow occurring around a rotor blade. Thus, a rigorous analysis including unsteady and three-dimensional effects, is necessary for the calculation of the blade bound circulation using inflow.

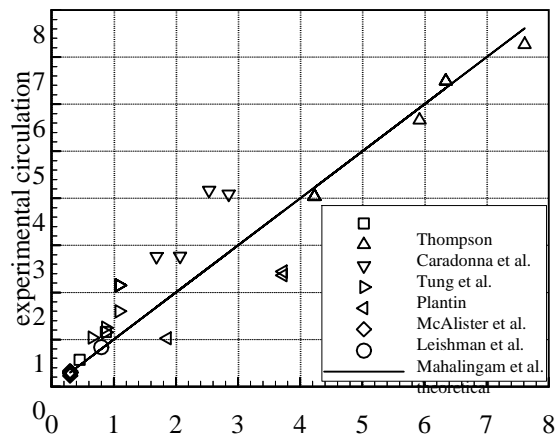


Figure 13: Correlation of several experiments with theoretical tip-vortex circulation

Effect of aspect ratio on the measured circulation

Figure 14 shows the effect of aspect ratio on the normalized circulation. This indicates that as the aspect ratio increases more of vortex sheet rolls into the tip-vortex. The tip-vortex circulation will be equal to the maximum bound circulation only when the aspect ratios are very large. However, about 80% of the peak bound circulation is attained at an aspect ratio of 5. Above that circulation is fairly insensitive to changes in aspect ratio. Since most of the tests compared here have been performed with wings of aspect ratio 5 or higher, we can assume that aspect ratio does not have a significant role to play in determining the circulation measured in these tests.

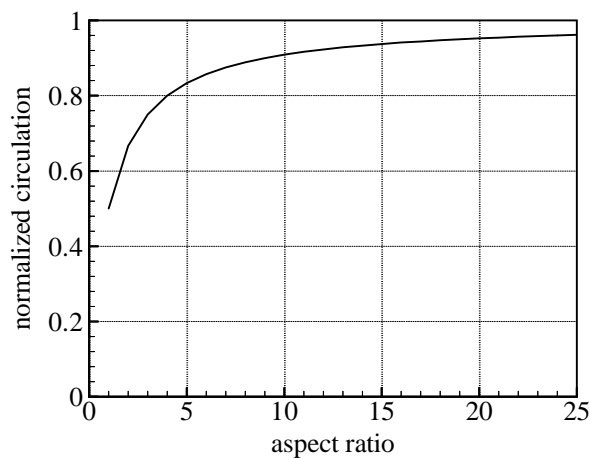


Figure 14: Effect of aspect ratio in the trailed tip-vortex circulation for rotary wings

CONCLUSIONS

1. An expression for the tip-vortex core circulation has been obtained in terms of AR, Reynolds number and geometric angle of attack.
2. Spanwise velocity of over 30% of the freestream speed is measured on the pressure side of the blade.
3. Peak axial and circumferential velocities during vortex formation are of comparable magnitude.
4. Core axial velocity approaches the blade tip speed at origin. Core circumferential velocities stay constant during the formation process.
5. In the near wake the core axial velocity exhibits a periodic variation suggesting the presence of discretely rolled up vortices around the vortex.
6. Vortex circulation measurements indicate that the peak bound circulation is over-predicted under the usual assumptions.

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