

Nonlinear aeroelastic response - Analyses and experiments

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NONLINEAR AEROELASTIC RESPONSE - ANALYSES AND EXPERIMENTS

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Abstract

The author examines nonlinear aeroelastic behavior. The effort includes both analyses and experiments. The research extends the effort of several recent investigations which address freeplay or piecewise nonlinearities in aeroelastic systems; however, in the studies described herein, the author addresses continuous nonlinearities such as those found in systems which exhibit softening or hardening structural stiffness effects. The author describes a unique flutter test apparatus designed to permit experimental investigations of prescribed nonlinear response.

Background

Aeroelasticity is traditionally considered to be the dynamical phenomena resulting from the interaction of structural, inertial, and aerodynamic forces. Classical bending-torsion flutter can be described as the coalescing of two or more vibration modes and is the type of flutter studied herein. A wing will vibrate with flexural (bending) and torsional (pitching) components. The frequency at which these vibrations occur may depend upon the airspeed (and other flow conditions such as the Mach number). The fundamental pitching and bending modes have distinct vibration frequencies at speeds lower than the critical flutter speed. As the velocity is increased, the frequencies of the pitching and bending motions may coalesce at a critical velocity resulting in an aerodynamic feedback. This may lead to divergent oscillations which will eventually cause structural failure.

Nomenclature

a	= nondimensional distance from the midchord to the elastic axis
b	= semichord of wing
C	= Theodorsen's function
C_y, C_α	= structural damping coefficients in plunge and pitch
I_x	= mass moment of inertia of the wing about the elastic axis
K_y, K_α	= structural spring constants in plunge and pitch
m	= mass of the wing
r	= distance between the elastic axis and the center of mass
V	= freestream velocity
y, α	= displacement coordinates in plunge and pitch
ϕ	= static angle of attack
ρ	= density of air
ξ, ζ	= nonlinear stiffening parameter in plunge and pitch

Conventional methods of examining aeroelastic behavior have relied on a linear approximation of the governing equations of the flowfield and/or the structure; however, aerospace systems inherently contain structural and aerodynamic nonlinearities^{1,2}. These nonlinearities result from unsteady aerodynamic sources, large strain-displacement conditions, and partial loss of structural or control integrity. These systems may exhibit nonlinear dynamic response characteristics such as limit cycle oscillations, internal resonances, and chaotic motion. Nonlinear theory describes an internal resonance phenomena in which a maximum exchange of energy occurs between the separate degrees of freedom if the higher modal frequencies are integer multiples of the lower modal frequencies. The important

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characteristic of this phenomena is that excitation of one mode will result in a response in another degree of freedom - a phenomena unexplained with the linearized equations of motion. Cole³ measured an unexpected response while performing wind tunnel experiments intended to identify aeroelastic stability. Cole used analytical methods based on linear theory and the analysis suggested higher dynamic pressures for aeroelastic flutter than those pressures experimentally measured. In Cole's experiment, the frequency of the second bending mode was twice the frequency of the first torsional mode. The signature of the response suggests that the inaccurate predictions are attributed to limitations arising from the use of linear theory.

Woolston, et al⁴, realized the importance of nonlinear effects on the flutter characteristics in 1956. They investigated nonlinearities such as freeplay, hysteresis and cubic stiffness effects. The results showed that the flutter stability was dependent upon the initial conditions. More specifically, an increase in the initial displacements reduced the flutter velocity. Limit cycle oscillations which transition to divergent flutter regimes occurred in the freeplay and hysteresis cases but only limit cycle oscillations occurred for cubic hardening curves.

Breitbach⁵ explained that even though linear theory flutter solutions can be a useful tool for flutter clearance, a significant number of cases suffer from poor agreement between analysis and test results. Many of these disagreements were traced to structural nonlinearities. Breitbach^{5,6,7} focused on nonlinear control surface effects such as hysteresis, freeplay and spring tab effects on the control system. Breitbach investigated the solution of the nonlinear control surface flutter equations in the frequency domain to provide a solution method which can be applied to control systems with multiple concentrated nonlinearities. He concluded that including only damping had a stabilizing effect on the flutter boundary while including both trim stiffness and damping had a destabilizing effect.

Farmer⁸ performed experimental flutter measurements. Farmer developed a two-degree-of-freedom rigid wing mount system to experimentally test flutter boundaries. This flexible mounting system provides linearly constrained pitch and plunge motions of the model. NASA⁹ is

using this mount system to acquire measured dynamic instability and corresponding pressure data that will be used to develop and evaluate aeroelastic computational fluid dynamic codes.

Our studies use a nonlinear aeroelastic model with two degrees of freedom. Similar models¹⁰⁻¹⁴ have been examined and provide a basis for comparison; however, the studies described herein implement continuous, nonlinear structural stiffening responses which represent a progression of the recent research in piecewise linear models (Fig. 1). Lee and LeBlanc¹⁰ performed analytical investigations of systems with cubic nonlinearities. Their work provides trends which relate the nonlinear response to variations in the initial conditions and nonlinear parameters. Lee, et al.¹⁰ investigate the effects of varying physical system parameters on the flutter envelope. The major trend found from this work is that spring hardening effects did not experience aeroelastic flutter, only limit-cycle behavior. As the velocity of the flowfield is increased, the amplitude of the limit-cycle oscillation increases and less time is required to reach the limit-cycle motion. Another important trend identified is that as the mass ratio is increased, the amplitude of the limit-cycle oscillation increases. Our approach permits us to investigate this hardening behavior experimentally.

Lee and Desrochers¹¹ analytically investigated freeplay and preload nonlinearities. An interesting result is that limit-cycle behavior appears in regions of otherwise damped oscillations. It is unknown if this can occur in continuously nonlinear structures, but is an area which deserves examination. Chaotic response is experimentally examined by Hauenstein¹² These results indicate transitions from limit-cycle oscillations to chaotic responses in freeplay nonlinearities. Chaotic response is analytically investigated by Price¹³ for continuous, nonlinear stiffness. Price¹³ demonstrated that chaos can occur in continuously nonlinear systems and showed the importance in identifying these possible transitions. Price, et al.¹⁴ also investigated free-play nonlinearities in pitch and identified limit cycle regions below the linear flutter speed and the existence of chaotic motion for cases of structural preload.

Theory

The aeroelastic system is modeled as a wing limited to motion in two degrees of freedom. As illustrated in Fig. 2, a rigid wing section is mounted to a support system which permits plunge and pitch motion. The equations of motion are

$$m\ddot{y} + mr \cos(\alpha + \phi)\ddot{\alpha} - mr \sin(\alpha + \phi)\dot{\alpha}^2 + c_y\dot{y} + K_y(y + \xi y^3) = -L$$

$$mr \cos(\alpha + \phi)\ddot{y} + I_e\ddot{\alpha} + c_\alpha\dot{\alpha} + K_\alpha(\alpha + \zeta\alpha^3) = M$$

where L and M are the aerodynamic loads,

$$L = L(\dot{y}, \dot{y}, \ddot{\alpha}, \dot{\alpha}, \alpha, q, t)$$

$$M = M(\dot{y}, \dot{y}, \ddot{\alpha}, \dot{\alpha}, \alpha, q, t)$$

and where the overdots represent time derivatives, y and α are the plunge and pitch coordinates, C_y and C_α are the damping coefficients for the support structure, K_y and K_α are the spring constants, ξ and ζ are nonlinear parameters, m is the mass, I_e is the mass moment of inertia about the elastic axis, r is the distance between the elastic axis and the center of mass, and ϕ is the static angle of attack. These equations appear to be uncoupled for the case when $r = 0$; yet, it is important to note that the aerodynamic loads are motion dependent and, therefore, couple the system. These equations reduce to the well known linear equations.

Herein, the aerodynamic lift and moment are modeled by the unsteady aerodynamic theory of Theodorsen¹⁵:

$$L = \pi\rho b^2(\dot{y} + V\dot{\alpha} - ba\ddot{\alpha}) + 2\pi\rho VCb(\dot{y} + V\alpha + b(\frac{1}{2} - a)\dot{\alpha})$$

$$M = -\pi\rho b^3(-a\ddot{y} + (\frac{1}{2} - a)V\dot{\alpha} + (\frac{1}{8} + a^2)b\ddot{\alpha}) + 2\pi\rho VCb^2(\frac{1}{2} + a)(\dot{y} + V\alpha + b(\frac{1}{2} - a)\dot{\alpha})$$

where b is the semichord, a is the nondimensional distance from the midchord to the elastic axis, V is

the freestream velocity, ρ is the density of air, and C is Theodorsen's function.

Experiment

The two degree-of-freedom analytical model has been used as the basis for the development of the experimental test apparatus (Fig. 3). The apparatus simulates aeroelastic bending and torsion behavior of a wing. The plunge degree of freedom is simulated by a carriage which translates on linear bearings. Two rotational bearings are mounted on this carriage and permit a pitch motion independent of the plunge motion. The pitch and plunge motions are constrained by springs (Fig. 4). Protective constraints limit the amplitude of motion and prevent damage from divergent flutter. Typically, only the mass of the wing is considered in the analysis. However, as a consequence of the support design, it is necessary to include the total mass -- wing and support structure -- in the analysis. In addition, the moment of inertia does not include the total mass since a portion of the system cannot pitch. This mass couples the motion of the system.

The structural stiffness response of the experiment is governed by a pair of cams. These cams are designed to permit either linear or tailored nonlinear stiffness response. The shape of each cam dictates the nature of the nonlinearity; thus, with this approach, these cams provide a large family of prescribed possibilities. The mechanism is mounted such that the wing is vertical (Fig. 5). The mounting position of the mechanism reduces the influence of the weight and minimizes undesired friction on the linear bearings. Our current investigations utilize the subsonic 2' x 3' wind tunnel at Texas A&M University; this facility provides speeds up to 46 m/sec (150 ft/sec).

The advantages of this device are numerous. One advantage is that the flutter instability can be investigated without wing failure. Thus, instead of approaching the instability boundary and extrapolating to predict its location, the actual flutter conditions can be achieved. Continuous, nonlinear restoring forces in both degrees of freedom may be investigated. This

apparatus compliments the type of research being conducted by others such as NASA's Benchmark Aeroelastic Models program. Farmer⁸ has tested a mount system limited to linear motion; his results will provide a source for comparison. Many investigations previously discussed have examined freeplay, deadband, and/or piecewise nonlinearities, but few have examined continuous nonlinear restoring forces analytically, and no experimental methods are available. These nonlinearities do exist¹. The device also provides much flexibility. The eccentricity of the aerodynamic center with respect to the elastic axis can be easily modified to experimentally determine its effect on the flutter conditions. The mass of the system, the moment of inertia of the wing, stiffness characteristics and the wing itself are parameters of the investigation.

Experimental flutter is determined by tracking the frequency content of acquired acceleration signals. Response is measured with accelerometers mounted on each degree of freedom. Tests are conducted by setting a freestream velocity and initial conditions, then releasing the structure. The acceleration signals are acquired using an A/D board and the frequency content of the signals is analyzed using Fast Fourier Transforms (FFT). The results are examined to determine the characteristics of the response before moving on to the next point.

Results

The analytical model and supporting experiment allow us to examine nonlinear response characteristics. We generalize the analysis of Lee, et. al and the experiments of Cole but will focus on the possible existence of nonlinear pathological response in one series of investigations. The existence of internal resonances will be investigated. The torsion and bending modes which might couple due to aeroelastic effects will be tuned such that the frequencies are almost integer multiples. These structural frequencies depend upon the dynamic pressure. Thus, even though these frequencies do not satisfy the integer relationship initially, these frequencies will shift due to an increase in dynamic pressure such that this integer multiple relationship will be satisfied. Consequently,

excitation is expected to occur due to the nonlinear coupling, similar to Cole's experiment.

Limit cycle behavior will be examined in the vicinity of the transition to flutter. In the analysis, regions of decaying amplitude oscillations are encountered which transition to limit cycle oscillations as the velocity increases. These transition velocities have been shown by others to depend upon the initial conditions. Regions of limit cycle instabilities are expected to occur experimentally.

In our current studies the nonlinear cams are tailored to provide specified cubic response behavior. The cams provide nonlinear stiffening. Initially, the coefficients of the nonlinear system were prescribed as $K_y = 2863 \text{ N/m}$, $\xi = 257$, $K_\alpha = 2.57 \text{ N-m/rad}$ and $\zeta = 3.42$. These nonlinear coefficients did not provide satisfactory nonlinear effects in the experiments. Thus, increased nonlinear stiffening, with coefficients of $\xi = 1047$ and $\zeta = 11.7$, will be introduced.

We examine sensitivity to system parameters and determine their effect on limit cycle and flutter boundaries. The elastic axis is located at 30% of the chord and the center of gravity is located at 41% of the chord. The natural frequencies are 1.96 Hz for the pitch degree of freedom and 2.77 Hz for the plunge degree of freedom. Predicted and experimental values are compared. Linear analysis indicates flutter will occur at a velocity of 16.5 m/s and a frequency of 2.56 Hz. The predicted flutter velocity and frequency for several positions of the elastic axis are presented in Fig. 6. The results in Fig. 6 show that flutter is not possible with the elastic axis located aft of 33% of the chord. This boundary agrees with general theory (see Pines¹⁶) which suggests that flutter will not occur when the center of gravity of the wing is placed forward of the elastic axis. A minimum flutter speed is reached when the elastic axis is located at 10% of the chord.

The pitch and plunge mechanism was tested with the elastic axis at 30% of the chord. The experiment had a linear pitch cam and a nonlinear plunge cam. The predicted flutter speed of 16.5 m/s was achieved with the linear model. Figure 7 shows the frequency content measured during our experimental investigation. Two cases

are shown -- below the critical velocity and at the critical velocity. The frequencies are shown to coalesce at a single aeroelastic frequency of 3.1 Hz.

An investigation was conducted with nonlinear hardening effects in both degrees of freedom. The response of the experimental system is subject to nonlinearities in aerodynamics and structural dynamics. It is extremely difficult to locate and predict when pathological responses, such as aeroelastic flutter, will occur. In Fig. 8, time-frequency decomposition is performed using the Fast Fourier Transforms (FFT) to characterize the transition to the instabilities. A horizontal slice of this view indicates the vibration frequency peaks which are shifting throughout the time history of the response. These results show the progression towards coalescence of the frequencies for increasing freestream velocity. The frequencies have coalesced at a freestream velocity of 16 m/s. The FFT of the measured response is normalized to the peak amplitude for each velocity.

The nonlinear aeroelastic response is simulated by solving the governing differential equations with Mathematica[®]. Theodorsen's function depends upon the reduced frequency (defined as $k = \omega b/V$). Herein, $C(k)$ is set to unity which suggests low reduced frequency motion. Structural damping is also neglected. Figure 9 depicts the plunge and pitch response for velocities less than the critical velocity and linear structural stiffening. Little coupling is evident in the analyses of the plunge motion, while there is significant coupling indicated by the pitch oscillations. This is due to the relative size of the masses in the support design. The motion of the small pitching mass which couples the system has significantly less ability to effect the motion of the larger plunge mass which cannot pitch. The position plot indicates coupled motion of two distinct frequencies which are shown on the FFT graph to have frequencies of 1.8 and 2.7 Hz. Evidence of coupled motion is further supported by the phase portrait which also shows two period motion. Figure 10 shows the response for the linear system at the critical flutter velocity. It can be seen that the motion is slowly diverging from both position graphs and that the frequencies have coalesced to 2.75 Hz in each degree of freedom. The phase portraits in this case indicate an expanding spiral for both motions.

Several nonlinear spring cases were tested to determine the effects of increased spring hardening. Figures 11 and 12 show the effects of increased hardening in the nonlinear springs at the linear critical flutter velocity. The limit cycle amplitude is reduced as the hardening is increased. Figure 11 indicates pitch and plunge limit cycle amplitudes at 0.2 radians and 0.03 meters respectively, both with frequency content at 2.9 Hz. The limit cycle motions are represented in the phase plane by the circular trajectories which are continually repeated after a given time. Figure 12 shows the pitch and plunge limit cycle amplitudes to be reduced to approximately half the initial amplitude for a 350% and 260% increase in spring nonlinearity respectively.

Conclusion

This paper presents investigations of nonlinear aeroelastic phenomena and describes a nonlinear flutter apparatus designed to permit 'tailored' studies of aeroelastic behavior. The major goals of this research are to accurately simulate, analyze, and predict nonlinear flutter response. Analysis and experimental results have been presented and show good agreement.

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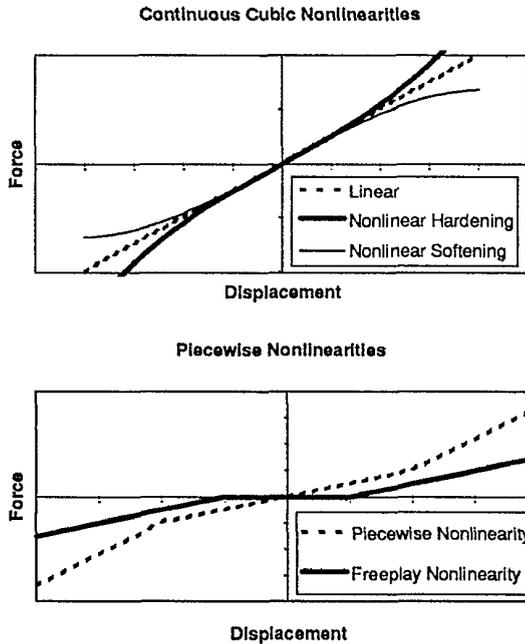


Fig.1 Continuous, nonlinear structural stiffness responses represent a progression of the recent research in piecewise nonlinear models.

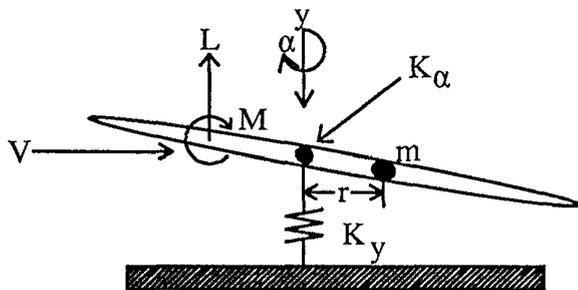


Fig. 2. The aeroelastic system is represented by a rigid wing attached to a flexible support which permits two degree-of-freedom motion.

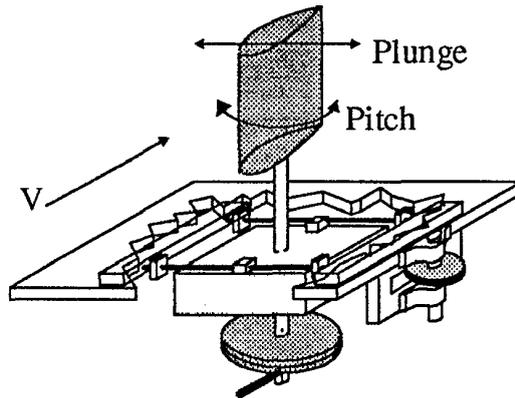


Fig 3. The flutter apparatus allows motion in pitch and plunge.

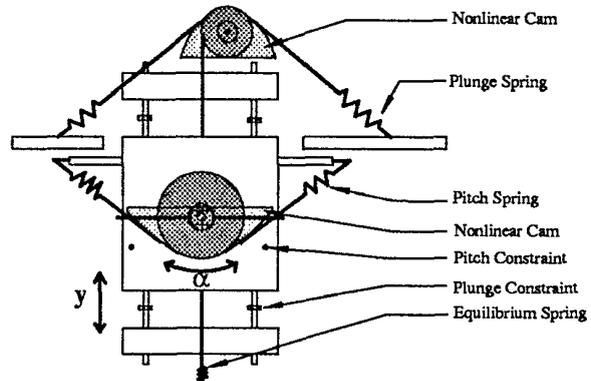


Fig. 4. The flutter apparatus is designed to provide tailored nonlinear response experiments. Nonlinear pitch and plunge response is introduced through two cams with prescribed shapes.

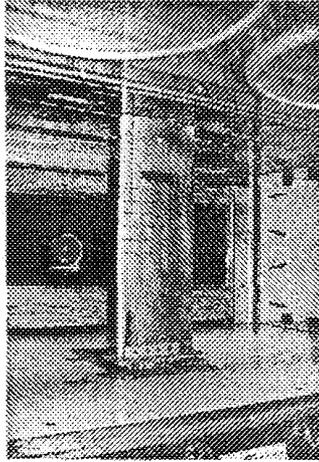


Fig. 5. The flutter apparatus and wing are shown within the test section at the 2' x 3' Low Speed Wind Tunnel at Texas A&M University.

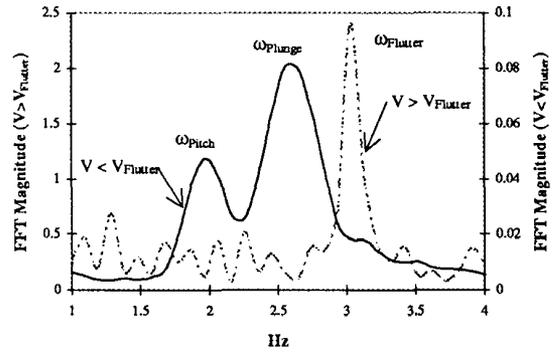


Fig. 7. Response of the system to the pitch and plunge motions is shown. Initially, two separate responses are observed, followed by a transition to a common aeroelastic frequency.

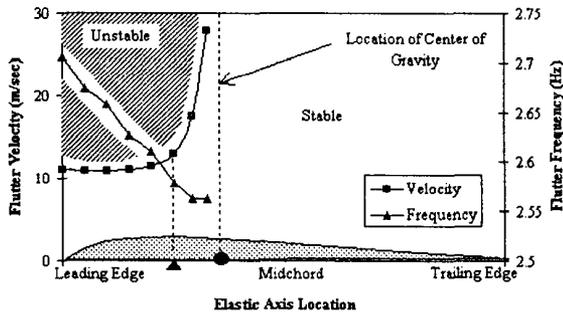


Fig. 6. Flutter velocity and frequency are shown for several locations of the elastic axis.

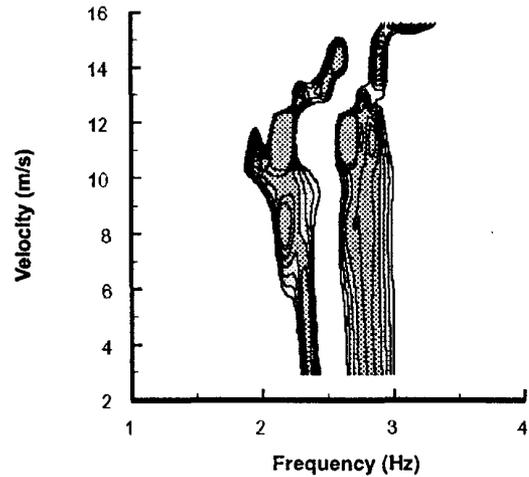


Fig. 8. The system frequencies depend upon the presence of aerodynamic loads. A characteristic of flutter is the coalescence of frequencies. The peak amplitudes are shown for increasing freestream velocity.

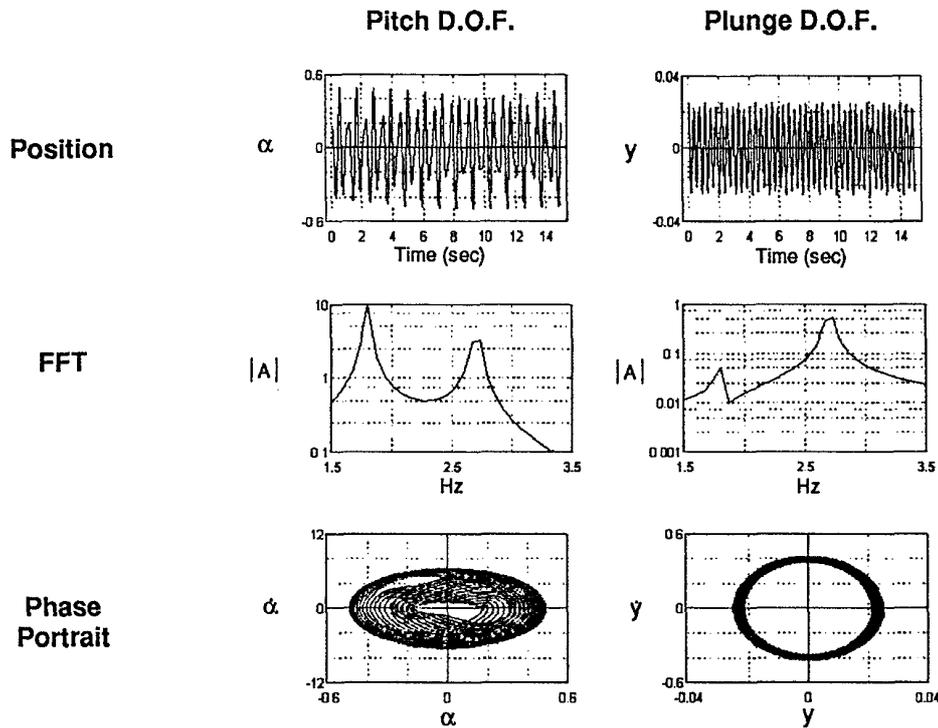


Fig.9. Linear response is predicted for subcritical velocities.

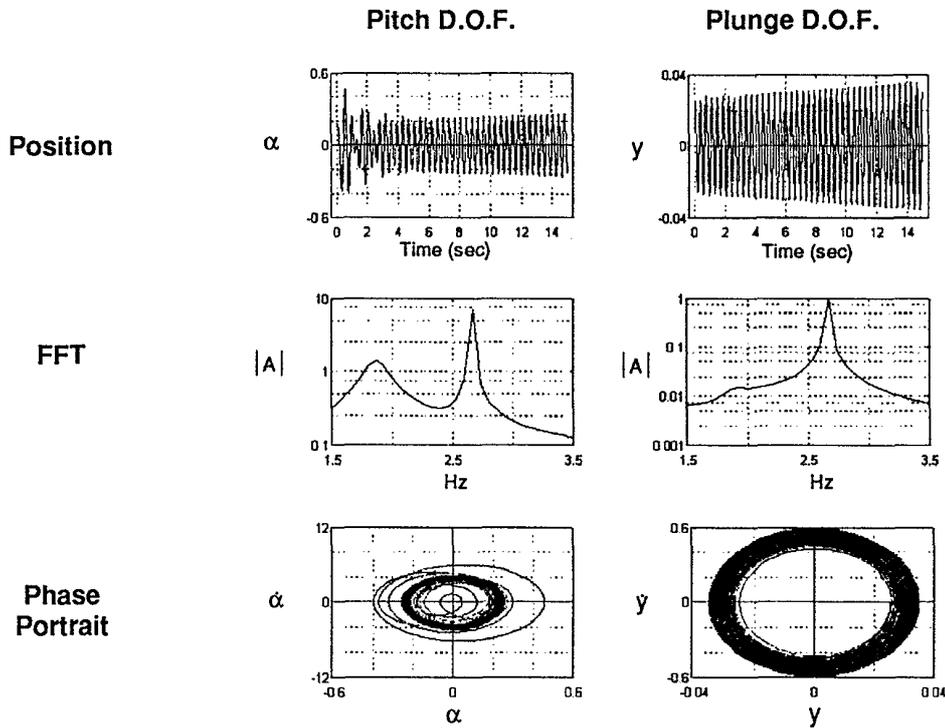


Fig.10. Linear response is predicted at critical velocities.

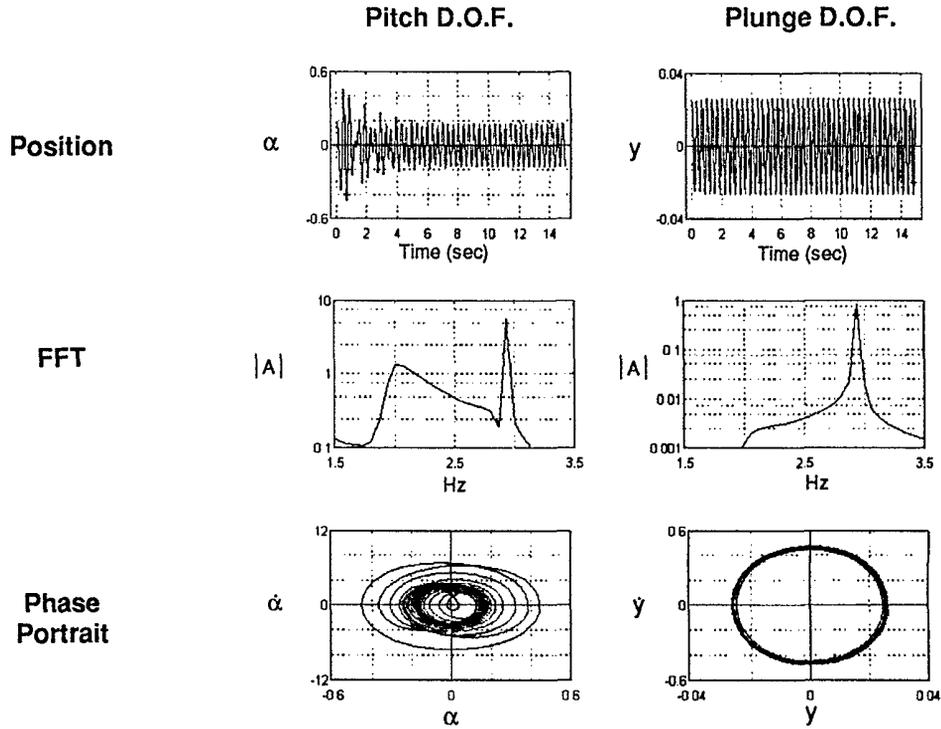


Fig.11. Nonlinear response is predicted at critical velocities ($\xi=400, \zeta=3.42$).

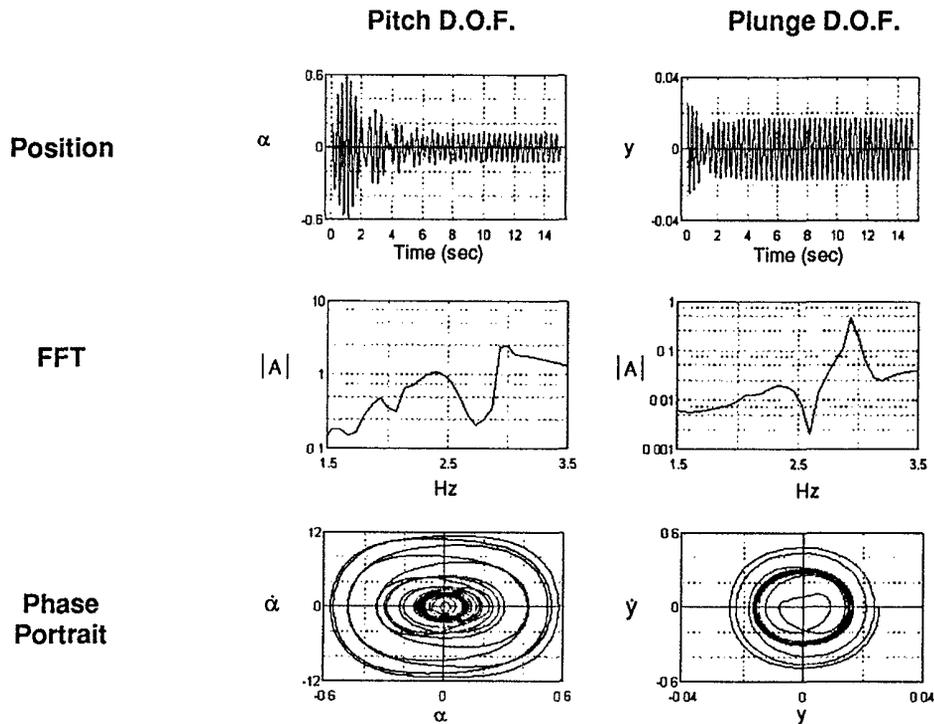


Fig.12. Nonlinear response is predicted at critical velocities ($\xi=1047, \zeta=11.7$).