

Status of Unsteady Aerodynamic Prediction for Flutter of High-Performance Aircraft

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I. Introduction

THIS paper presents a summary of the state of the art of unsteady aerodynamics as applied to production flutter analyses for high-performance military aircraft. The data presented here are based to a large extent on the author's personal experience, and although every attempt has been made to provide a complete account, success may not have been achieved in all cases. The requirements for flutter analyses and, therefore, the need for unsteady aerodynamic methods are documented in Mil-8870. However, the exact form and tools to be used for these analyses are not spelled out by that document. Modified strip theory was the mainstay for just about all production flutter analyses 30 years ago. Today this role is largely filled by the doublet-lattice code for subsonic analyses with multiple codes being used for the supersonic speed regime. Whereas computational fluid dynamics has made a revolutionary impact on steady-state aerodynamic analyses, the same cannot be said in the unsteady aerodynamics field, especially when one considers production flutter analyses. Work to address these issues is under way. However, a production capability is not anticipated anytime soon.

High-performance military aircraft are usually flutter critical in the transonic speed range at high dynamic pressure. The unsteady aerodynamics required in this region is extremely complex, and the current tools, basically unsteady panel methods, may require corrections for accurate flutter predictions. Gross corrections are usually derived from transonic wind-tunnel flutter model testing. However, progress has been made using steady computational fluid dynamics as the basis for aerodynamic corrections.

Flutter analyses for high-performance aircraft are driven by a set of requirements that will ultimately affect the choice of the aerodynamic tools to be used, and discussion of these requirements is presented as a part of this presentation.

To understand the current state of the art of the current tool set, a discussion of some recent history of unsteady aerodynamics as applied to high-performance aircraft is also in order. This discussion begins with the transition from strip theory to the doublet-lattice method that took place in the late 1960s to early 1970s. The application of these methods to some of the aircraft that were designed at that time will provide some insight into the methodology and will serve as a framework for discussion of current methods and future developments.

Current production unsteady aerodynamics codes are based on panel methods. The capabilities present in these methods to model complex shapes represents one of the major advancements in this methodology. A second advancement is that the new codes are now formulated based on a unified analytical capability throughout the Mach range. Examples of these capabilities will be demonstrated.

Finally, based on the discussion of requirements for flutter analyses of high-performance aircraft, a description of the needs for the future is presented. It is hoped that these recommendations will provide guidance to those investigators working in the field of unsteady aerodynamics.

II. Background

A. Unsteady Aerodynamic Methods

Strip theory aerodynamics originated in the early 1940s,^{1,2} and this method was the primary aerodynamic tool for flutter analyses for many years. In 1966, Yates³ proposed modifying $C_{L\alpha}$ to account for finite span effects with the result being referred to as modified strip theory. This aerodynamic method, coupled with the normal mode approach, and the $V-g-\omega$ solution technique formed the basis for production flutter analyses in the late 1960s, and it was to this methodology that the author was first exposed to production flutter analyses.

B. Doublet Lattice

During the mid to late 1960s, a series of events⁴ led to the development of a remarkable unsteady aerodynamic tool, the doublet-lattice method. A complete description of these events is not warranted at this point. However, Ref. 4 is highly recommended to the interested reader. Note, in Ref. 4 it is stated that the development of the doublet-lattice method at one point was carried out under a bootlegged research and development time charge. It is interesting in this day and age of detailed planning that a good idea can often come from an under the table source. Perhaps there is a lesson that the flutter community could learn from this. Whereas Ref. 4 is the definitive history of the method, the definitive reference on the doublet-lattice method itself is usually cited as Ref. 5.

Further developments of the method were carried out at Douglas, Long Beach, California,⁶ and the product of this investigation was the H7WC code. This extension allowed the method to handle non-planar aerodynamic surfaces with bodies. In addition, this code



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produced aerodynamic influence coefficients (AICs), which was a feature introduced in earlier versions of the code (H7WA, isolated surfaces, and H7WB, nonplanar). AICs must be considered one of the most important contributions of the doublet-lattice method to production flutter analyses. When AICs was used, the cost of the doublet-lattice method was comparable to that obtained using modified strip theory, which was the industry standard at that time. However, a later version of the code, N5KA,⁷ did not retain this feature. The N5KA code also had the unfortunate feature that it used polynomials to represent the structural mode shapes. The result was that flutter speeds predicted with this code were dependent on the order of the polynomials that were selected. The cost of this code was also prohibitive for production use. These difficulties were later corrected in the N5KM version of the code.

More recently, Rodden et al. have continued to refine the doublet-lattice method.^{8,9} These enhancements were the replacement of the parabolic approximation of the numerators of the incremental kernels by quartics and improved approximation to the integrand in the integral in the kernel. These improvements were incorporated into a new code called N5KQ. Whereas N5KQ represents an improvement relative to its predecessors, care should be taken in mixing results from this code with earlier versions because results may be different.

In this short summary of the doublet-lattice method, one additional reference needs to be included.¹⁰ Reference 10 should be consulted for a summary of the mathematics of the double-lattice method.

The doublet-lattice method has stood the test of time. It has been in use for over 30 years and has become the standard by which other unsteady aerodynamic codes are judged. It is anticipated that, like some of the aircraft that it was used to design, the code will continue to be used for many years to come. Three features can be identified that are responsible for this longevity of use. The first, which should go with out saying, is that the code is accurate enough for production flutter analyses. Of course, the answers are not correct in the transonic speed range or when there is separation, etc. These shortcomings are well known, and corrections based on wind-tunnel flutter models can be applied to rectify this. The second feature, already mentioned, is that the method produces AICs. This feature allows the method to be cost competitive with simpler methods, such as modified strip theory, once the AICs are computed. The third feature is the method's ability to model fairly complex geometry. Lifting surfaces are simply paneled with a series of chordwise strips that are further subdivided into boxes. Bodies can be represented using slender-body theory and interference tubes. With these tools, aircraft of almost any shape can be represented. As an example, a wing-tip missile such as that carried by the F-16 or the F/A-18 can be modeled in a straightforward fashion.

Other unsteady aerodynamic theories have been proposed over the past 40 years, but none seems to have achieved the popularity of doublet lattice. For example, in the subsonic speed range, the kernel function method¹¹⁻¹³ was an initial competitor to the doublet-lattice method. However, this method required the use of pressure modes, and these were unique to the configuration that was being analyzed. The cost of the method relative to strip theory or to doublet lattice once AICs were computed, was high. As a result, this method lost favor for production flutter analyses.

C. Supersonic Methods

Whereas doublet lattice appears to be the clear choice for subsonic unsteady aerodynamics for production flutter analyses, no clear winner has emerged for the supersonic-speed regime. There have been various methods for implementing supersonic lifting surface theory. The most traditional is the Mach box method.¹⁴⁻¹⁶ However, this method is very awkward to use. The problems are well known: For subsonic leading edges, the downwash in the diaphragm region ahead of the leading edge must be determined; the leading edge, which locates a numerical singularity, is approximated by a stair case of box corners; and a large number of boxes are required for low supersonic Mach numbers. In addition, the method is not recommended below a Mach number of 1.414. However, reasonable solu-

tions as low as 1.2 have been obtained. The method does not compute AICs, and hence, the cost of the method is high relative to doublet-lattice or modified strip theory, which were popular at the time.

An approach proposed by Burkhart¹⁷ extended Evvard's steady supersonic aerodynamic theory¹⁸ to the unsteady case. The approach makes use of a downwash correction scheme developed by Ferguson and Guderley¹⁹ to allow the theory to be applied to oscillatory wings. The implementation of the theory into a usable code is based on the use of integration regions on the wing that take the form of parallelograms. Because of this integration over parallelogram regions, the code was given the name supersonic parallelogram integration program (SPIP). Part of the integration over the wing is used to cancel the integration over the diaphragm region and evaluation of the downwash over that region is not required. The code was extended from the one defined in Ref. 17 to include leading- and trailing-edge control surfaces, and this code was used for all production supersonic flutter analyses for the F/A-18A. Burkhart¹⁷ received the award for best paper presented at the AIAA Structures, Structural Dynamics and Materials (SDM) meeting in 1980. This code had several faults. First, it did not produce AICs, and the code was expensive relative to doublet lattice. Second, the code was never extended to the multisurface case, and consequently, only an isolated surface could be analyzed. Third, the code had restrictions on its modeling capability. Specifically, for the F/A-18, the tip missile could not be modeled.

An additional method that appeared to offer promise as a supersonic code was the doublet-point method as proposed by Ueda and Dowell (subsonic)²⁰ and supersonic.²¹ This method had the added advantage that a single code could handle both the subsonic and supersonic Mach ranges. Eversman and Pitt developed a usable subsonic code.²² However, this code offered no real advantage over the doublet-lattice method. Additional work on the combined subsonic-supersonic version of the code resulted in Ref. 23. Although, it appeared that the analytical formulation indicated success, the work as documented in Ref. 23 did not produce a usable code.

Chen and Liu²⁴ make the point that as late as 1985 a good supersonic unsteady aerodynamics code was not available. "After 35 years of supersonic flight, a need still exists for a reliable prediction of unsteady airloads on interfering lifting surface configurations in supersonic flow. Although requirements for subsonic flutter analysis of interfering configurations are satisfied by the doublet-lattice method, an equally effective supersonic method has been lacking." Chen and Liu go on to describe a harmonic gradient method for unsteady supersonic flows calculations. Chen and Liu begin by categorizing supersonic lifting surface methods into three types: those that adopt 1) the velocity potential, 2) the acceleration potential, and 3) the gradient of the velocity potential as the dependent variable. Typical of the first is the Mach box method, already discussed, whose shortcomings include that the velocity potential must be determined off the planform in the diaphragm region. Typical of the second category are the supersonic doublet-lattice method²⁵ and the kernel function method.^{26,27} None of these methods appear to have resulted in a production flutter analysis tool. The third category has produced the potential-gradient method.²⁸ This method has the ability to model an exact idealization of the planform without any need for assumed pressure modes and can use a grid similar to what would be generated for a doublet-lattice model. However, the method shows some inaccuracies at high-reduced frequencies and may require a large number of boxes to achieve converged results.

The development of the harmonic-gradient method²⁴ was motivated by the aeroelastic requirements set by the configurations of modern fighter aircraft. For these aircraft, complex canard-wing, wing plus tip missile, wing-tail, and vertical tail-stabilator interactions require complex paneling to predict the flutter boundaries correctly. The harmonic-gradient method was shown to not only reduce the number of panels required for these complex configurations but also yielded improved accuracy in all cases evaluated. The harmonic-gradient method was further validated by additional studies where it was compared to other supersonic aerodynamic theories and to experiment.²⁹ In addition, improvements to the method were also described relative to the code that was initially presented in Ref. 24. The main improvement was the reformulation of the code

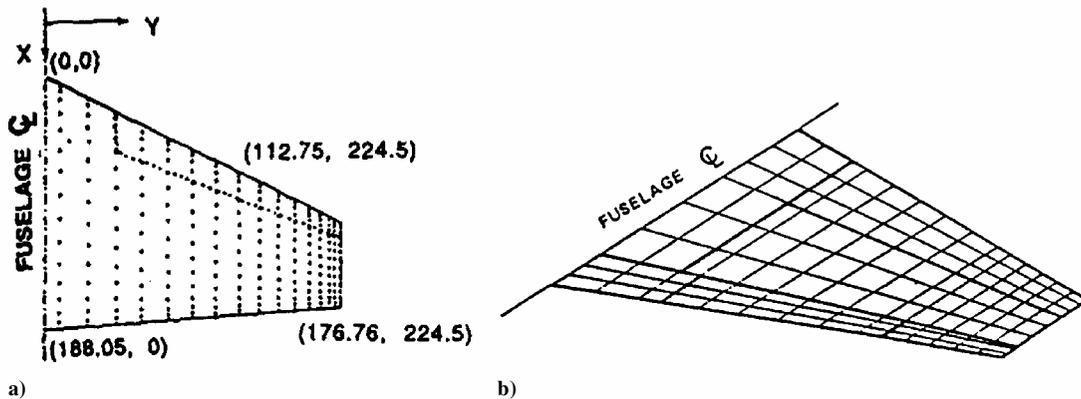


Fig. 1 Aerodynamic modeling of F-18 wing.

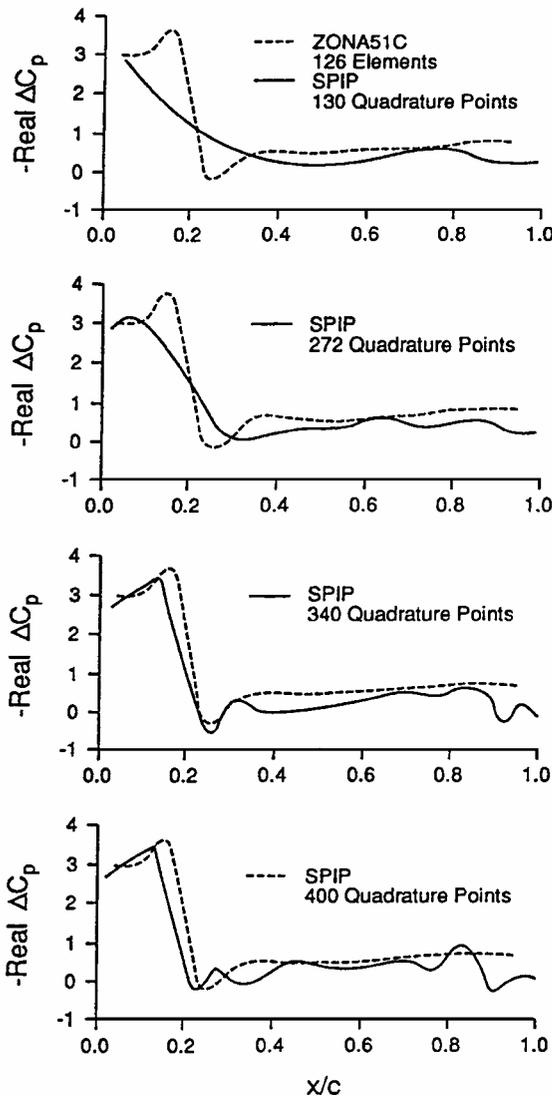


Fig. 2 Effect of panels on computed pressures for an F-18 wing with an oscillating leading-edge flap at 58.8% span, $M = 1.1$ and $k = 4.0$.

from a velocity potential formulation to an acceleration potential formulation. This new code was called ZONA51C.

Some of the results from Ref. 29 are reproduced here to demonstrate the effectiveness of the ZONA51C code. For the first case, the ZONA51C code is compared to results obtained using the SPIP code that was described in Ref. 17. The SPIP code uses numerical integration, and the accuracy of the code depends on the number of Gauss quadrature points used. Figure 1 shows the planform and modeling for the F/A-18 wing for both codes. The paneling for the ZONA51C code remained at 128 boxes. Figure 2 presents data for

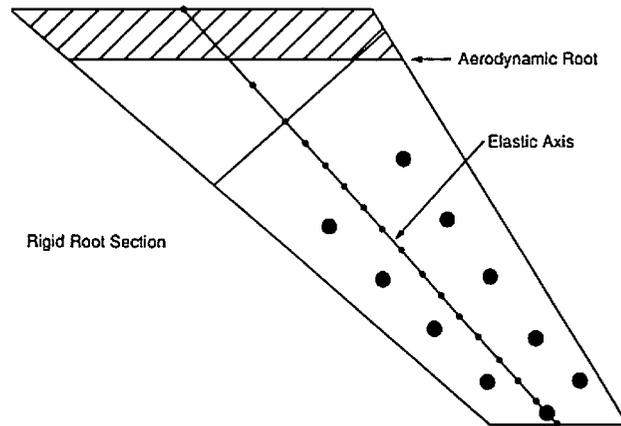


Fig. 3 U.S. Air Force Wright Aeronautical Laboratories. AFWAL wing flutter model with lumped mass NASTRAN representations: ●, structure grid points and ●, lumped masses.

the real value of ΔC_p as a function of x/c . The data is 58.8% span. It can be seen that the results from the SPIP code appear to converge to those predicted by ZONA51C.

In Ref. 29 a series of comparisons for flutter results was also presented. One example originally presented in Ref. 30 is reproduced here as Figs. 3 and 4. Figure 3 shows the planform of the model along with the elastic axis, structural grid points, and mass points. The comparison of flutter speeds as predicted by the ZONA51C and the constant pressure panel method (CPM)³¹ is given in Fig. 4. The data are presented for a Mach number of 1.081 with flutter speed and frequency plotted as a function of the number of panels in the model. As can be seen from the data, the results predicted by the ZONA51C are in far better agreement with the experimental results than those obtained from the CPM method. Note that the code in Ref. 31 is the code that is used in ASTROS³² for supersonic flutter analyses. This code has achieved an excellent reputation and is considered to be one of the important features of the ASTROS code.

Other cases to demonstrate the validity of the method are presented in Ref. 29. Space does not permit a complete review at this point. However, the results demonstrate the superiority of the ZONA51C code for supersonic flutter analyses.

An improved version of ZONA51C, ZONA51D, was incorporated into the flutter solution sequence of version 67 of MSC/NASTRAN.³³ This selection was viewed as a vote of confidence in the capabilities of the ZONA51 code.

In Ref. 34, an additional evaluation is provided of the accuracy and the utility of the ZONA51D code. The results, reproduced here as Fig. 5, show a comparison of the complex lift generated on the wing of a wing-canard configuration due to canard pitch. The data are presented as a function of reduced frequency for a range of 0–5.0 and a Mach number of 1.054. The results obtained by the ZONA51D code are in excellent agreement with those obtained by the National Aerospace Laboratory, the Netherlands and published in Ref. 34.

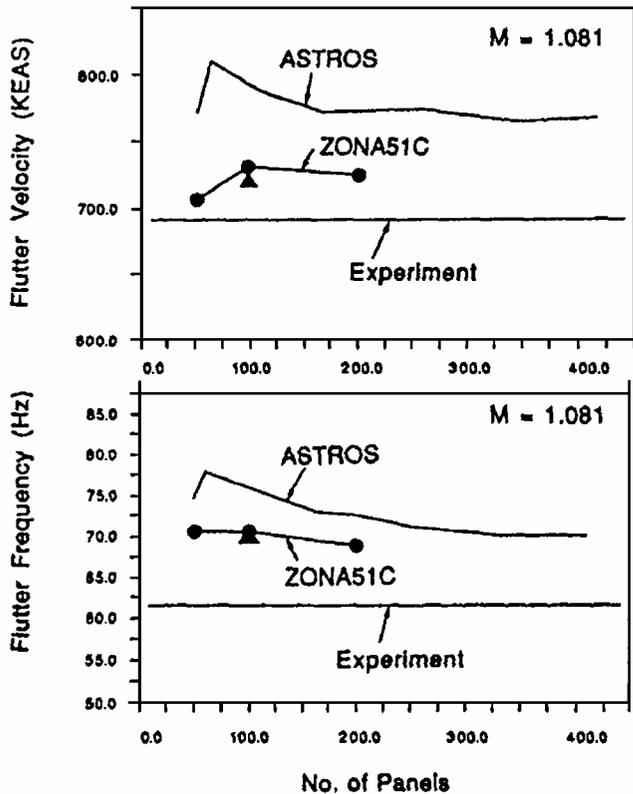


Fig. 4 Effects of panel numbers on flutter speed and frequency.

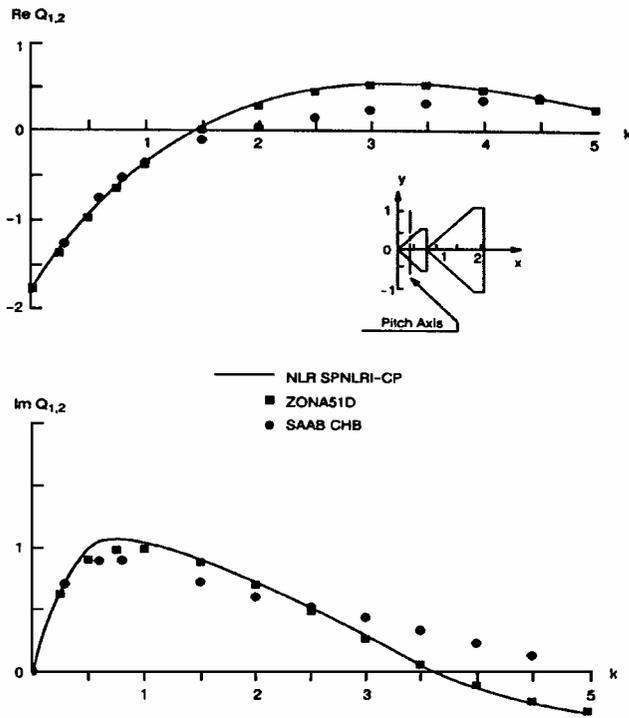


Fig. 5 Real and imaginary part of the lift on the main wing due to pitch of the canard, $M = 1.054$.

Whereas a comprehensive evaluation of all the supersonic methods so far described is not available, a comparison of flutter speed predictions for four of the methods has been made. This comparison was done using the F/A-18A horizontal tail, which is an all-moving surface with a relatively simple flutter mechanism, first bending coupling with the pitch-rotation mode. The pitch-rotation mode can be characterized as a rigid-body rotation about the spindle restrained by the actuator. To obtain a reasonably converged solution, 10 modes are required. The codes that were compared were Mach

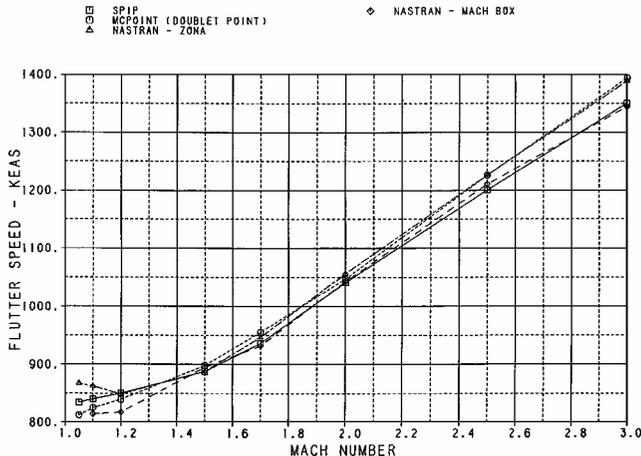


Fig. 6 Flutter speeds predicted by four supersonic aerodynamic codes.

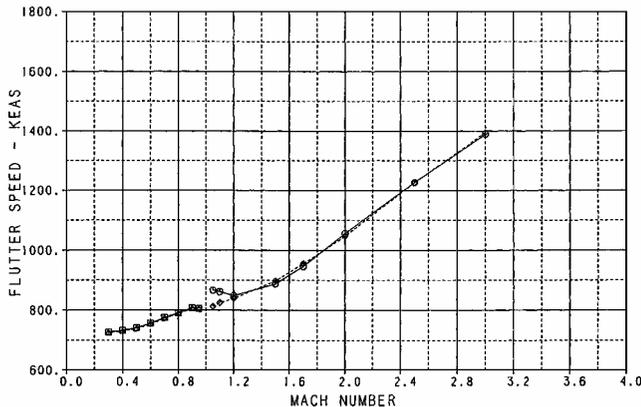


Fig. 7 Predicted flutter speed matchup with doublet lattice for two supersonic aerodynamics codes: \square , MSC (subsonic); \circ , MSC (supersonic); \triangle , MCPPoint (subsonic); and \diamond , MCPPoint (supersonic).

box, doublet point, SPIP, and the ZONA51 code as implemented in MSC/NASTRAN. The results, flutter speed as a function of Mach number, are presented in Fig. 6. As might be anticipated, the greatest disagreement in the results is obtained in the low supersonic Mach range, 1.05–1.4. At high Mach numbers, the four codes pair up into two groups, Mach box with the SPIP code and ZONA with the doublet-point method. With only these data it would be difficult to determine if one code is superior to any other. However, if the results from doublet lattice for the subsonic Mach range are added (Fig. 7), it can be seen that the doublet-point method does the best job of connecting up with those results.

Unlike the subsonic speed range, where doublet lattice has emerged as the code of choice, there is no clear winner for the best supersonic code. Indeed, each method has demonstrated elements of success. Each code has its proponents. Although every supersonic unsteady code has not been reviewed, it is believed those presented are sufficiently representative of what is available. In fact, based on requirements that will be shown in a later section, it will be concluded that the ZONA51 code is the best choice.

III. Unsteady Aerodynamic Applications

During the past 30 years, a number of high-performance military aircraft have been built, and a short history of some of them should be instructive for lessons learned. As described earlier, 30 years ago, modified strip theory was the main tool for production flutter analysis, and the new generation of unsteady aerodynamic codes was just being introduced.

A. F-15 Eagle

Figure 8 shows the F-15 Eagle Air Superiority Fighter. The time frame for the development of this aircraft, 1969 for the contract definition phase and 1970–1974 for the hardware development phase,³⁵



Fig. 8 F-15 Eagle Air Superiority Fighter.

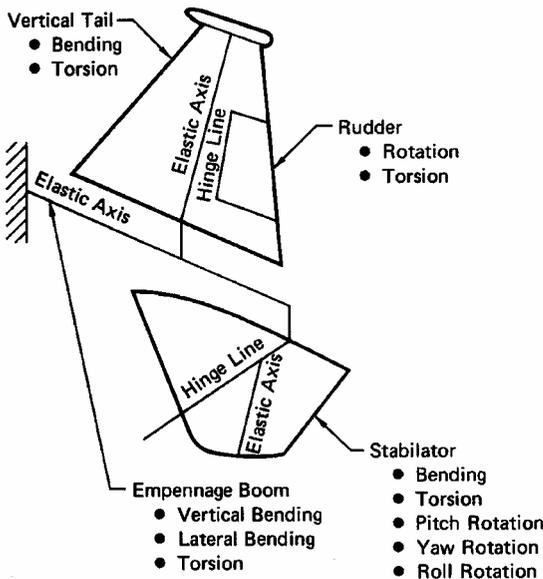


Fig. 9 F-15 empennage arrangement.

corresponds very closely with the formulation and development of the doublet-lattice method. One might expect that the doublet-lattice method played a significant role in the development of this aircraft, and this is indeed true. As shown in Ref. 35, the empennage was critical from a flutter standpoint, and considerable effort was expended to ensure that flutter did not occur inside the required speed range.

Achieving the required speed margins for the empennage was especially challenging. Both the vertical tail and the horizontal tail, rather than being mounted directly to the fuselage, were mounted on a flexible boom structure, which added additional degrees of freedom, as well as coupled the two surfaces both structurally and aerodynamically. A schematic of the empennage arrangement is shown in Fig. 9. The structural coupling that occurred because of the boom structure was relatively easy to account for. However, modeling the aerodynamic coupling or interference between the two surfaces with the standard strip theory was not possible.

If the aerodynamic interference were the only effect, there would not be much of a story because, in the final analysis, this effect was found to be minimal. It was not. Figure 10 shows a plot of flutter speed plotted as a function of stabilator rotation frequency. The key flutter mechanism for this surface involved coupling of the stabilator first bending mode with the pitch-rotation mode. Consequently, the rotation frequency of the surface on the spindle being restrained by the actuator was a key parameter. Figure 10 shows experimental flutter points as determined by wind-tunnel flutter model testing and results predicted using various aerodynamic theories. The key parameter here is the definition of the actuator stiffness required to preclude flutter. This stiffness requirement directly affected the weight of the actuator.

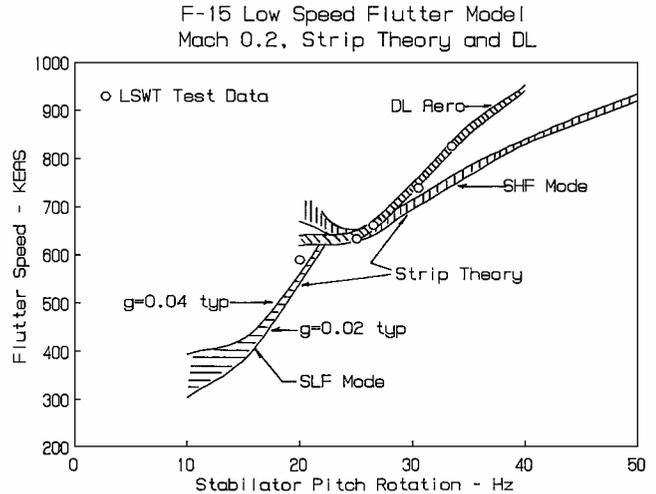


Fig. 10 Flutter speed predicted using doublet lattice and experimental data for the F-15 stabilator.

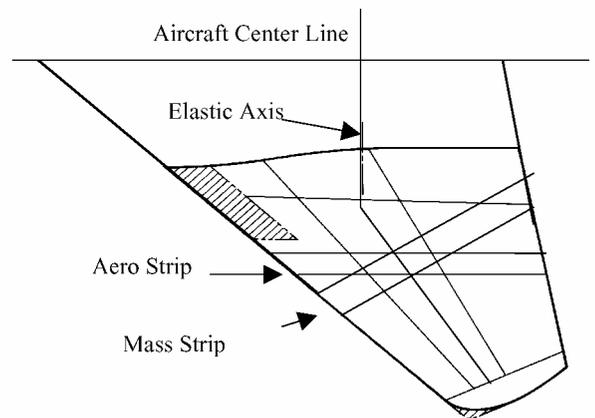


Fig. 11 F-15 stabilator modeling issues for doublet lattice.

Whereas modified strip theory could predict the flutter speed with minimum restraint stiffness, it could not correctly predict the trend. Although the doublet-lattice code is shown predicting the experimental data correctly, some interesting things were learned about doublet-lattice modeling in achieving this result.

Figure 11 shows two features of the stabilator planform. The first is that the moving part of the surface is connected to a fixed or pressure carryover region. In doublet-lattice modeling, the effective aspect ratio is very important, and the model must represent the surface in its correct position in space. In this case, the fuselage region from the stabilator root rib to the centerline of the aircraft must also be modeled. Although this effect may seem obvious today, 30 years ago it was not. Many people argued that the fuselage provided an effective end plate, and the centerline of the doublet-lattice model could be placed at the root rib. Such an approach leads to a very conservative estimate of the flutter speed.

The second effect shown in Fig. 11 is an effect called induced camber. The stabilator is a low aspect ratio swept surface with the structural mass strips oriented perpendicular to the swept elastic axis. However, the aerodynamic strips are oriented parallel to the airstream. The net result is that the effective angle of attack due to bending or torsion of the surface varies along the aerodynamic strip and this induced camber must be accounted for. Today, with a detailed finite element structural model and spline representations for the mode shapes, this would not be an issue. Fortunately, the doublet-lattice code being used (the H7WB code, similar to the H7WC code already mentioned but with out the body elements) could account for both of these effects, and the analytical results shown in Fig. 10 exactly bracket the experimental values if structural damping of 0.0 and 0.02 are introduced. This was a very important contribution to the design of the F-15.

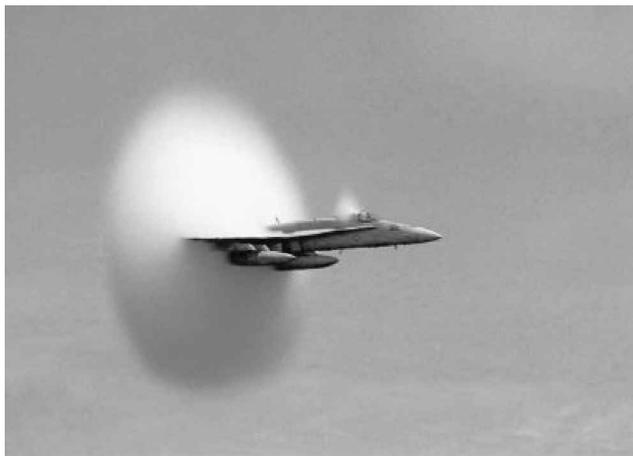


Fig. 12 F/A-18A Hornet.

Although these results are interesting, in Ref. 35 it is also shown that the final solution for flutter of the stabilator involves a leading-edge snag. The increase in flutter speed achieved with this modification was found from wind-tunnel flutter model testing and doublet lattice was not able to predict this flutter speed increase. More will be said on this subject in a later section.

Mach box, described earlier, was used for the supersonic flutter studies. In comparison to the doublet-lattice code, this code was difficult to use and could not account for interfering surfaces, the stabilator pressure carryover region, or induced camber. Because the code did not produce AICs, it was expensive to use. What was more difficult to interpret was that flutter speeds predicted by Mach box were inconsistent with those predicted by doublet lattice. In general, the contributions made by Mach box to the design of the F-15 were minimal. Flutter clearance in the supersonic speed range relied on strip theory corrected by wind-tunnel flutter model results.

B. F/A-18A Hornet

The Hornet (Fig. 12) was designed approximately 10 years after the F-15, and improvements in the unsteady aerodynamic tools that were used would be expected. This was true for the supersonic speed range but doublet lattice with some improvements remained as the tool for production flutter analyses for the subsonic speed range.

Whereas the empennage was the main challenge for the F-15, the wing required the most attention for the F/A-18. A description of the wing flutter issues is provided in Ref. 36. The thickness-to-chord ratio for the wing starts at 5% at the root, reduces to 3.5% at the fold, and remains constant from there to the wing tip. With a relatively thin wing such as this, it is very difficult to obtain the necessary torsional stiffness to preclude flutter. Whereas this contributed to the problem, the main issue³⁶ was the leading-edge flaps. These surfaces are by their nature divergence prone, and this was recognized early in the design process. What was not realized was that the divergent flap rotation mode was responsible for generating the wing critical flutter mechanism. Literally, the divergent flap mode drove the wing torsion mode into the bending mode, which caused the wing to flutter. This was the first time that this type of flutter mechanism had been observed with McDonnell aircraft.

This divergence-driven flutter mechanism imposed some new requirements on the flutter analyses. First, an adequate number of normal modes needed to be included such that the flap rotation modes were included in the analysis. This meant that between 30 and 40 modes were required, and this was more than double what was used for the majority of the F-15 flutter analyses. The second requirement was that the unsteady aerodynamic codes be able to compute accurately the aerodynamics for the leading-edge flap modes. The wing-tip missile generated the third requirement. As it turned out, aerodynamic modeling of the tip missile was a key issue. This was not well understood early in the program.

For the doublet-lattice code, neither of the first two requirements posed any real challenge. Once the role of the leading-edge flaps was



Fig. 13 AV-8B Harrier.

understood, the analyses were modified, and work continued. For the supersonic speed range, the Mach box code, as used on the F-15, was totally inadequate. Fortunately, the SPIP code described earlier had been developed and with some modification was able to handle the requirements imposed by the leading-edge flaps. Aerodynamic modeling of the wing-tip missile was an issue early in the program from the standpoint that the SPIP code did not have the capability to model it. One position that was considered was that both the subsonic and supersonic aerodynamic models should be consistent, and it was suggested that the tip missile should not be included in the doublet-lattice model. This was not done. However, the tip missile model that was used was based on reducing the missile and its launcher to the equivalent projected area.³⁷ Unfortunately, this model was found to be inadequate during flight flutter testing of the aircraft with external stores.

The progress in the 10 years from the time the F-15 was analyzed was an improved version of the doublet-lattice method and a new supersonic code. Although the SPIP code represented an improvement over Mach box, a better supersonic code was still desired.

C. AV-8B Harrier

This aircraft (Fig. 13) was designed during the late 1970s, roughly two years after the F/A-18A. Although the aircraft can go supersonic, its capabilities in this speed regime are limited. The challenge for this aircraft from a flutter standpoint resulted from the incorporation of a thick supercritical airfoil.³⁸ Because of this airfoil section, a large transonic flutter speed dip was found during wind-tunnel flutter model testing. In Ref. 38, a correction procedure was demonstrated that made use of a steady-state computational fluid dynamics solution to correct the unsteady doublet-lattice AICs. This procedure did an adequate job of predicting the observed dip. Fortunately, because of the thick airfoil section (high thickness-to-chord ratio), this wing had high torsional stiffness, which more than compensated for the reduction in flutter speed in the transonic Mach range. The lesson to be learned from this, however, is that for transonic flutter calculations a procedure for correcting the doublet-lattice results that does not depend on wind-tunnel flutter model testing can be used as an effective alternative to wind-tunnel flutter model testing.

D. F/A-18E Super Hornet

This aircraft (Fig. 14) was designed in the early 1990s, and lessons learned from the previous 20 years were available.

A detailed doublet-lattice model of the wing with tip missile developed using the N5KM version of the code was constructed and was used for flutter calculations. For the supersonic speed region, the doublet-point code was used. Although this code lacked the detailed modeling capabilities available in the N5KM doublet-lattice code, the code did have the ability to compute AICs. This contributed a significant cost reduction, which is important for production flutter analyses.

In summary, these four aircraft represent over 30 years of flutter analyses, and one fact stands out. The doublet-lattice code has been



Fig. 14 F/A-18E Super Hornet.

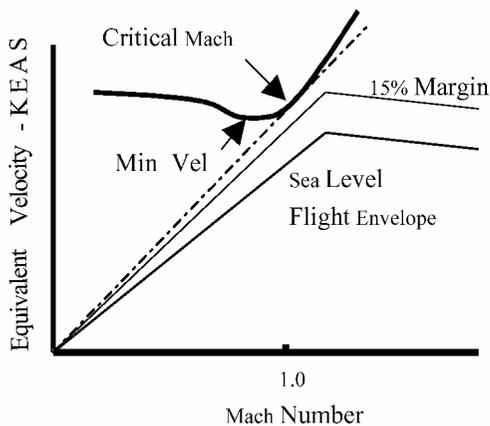


Fig. 15 Flight envelope and flutter boundary.

the mainstay of all of the subsonic flutter analyses. For the supersonic speed range, no single code has appeared that is comparable to doublet lattice.

IV. What Is Needed?

This section provides a short description of what is required in an unsteady aerodynamics code that is to be used for production flutter analyses of a high-performance aircraft. A survey and a history of unsteady aerodynamics codes for aeroelastic applications have already been presented, and it has been shown how these codes have contributed to the design and development of several high-performance aircraft over the past 30 years.

Figure 15 shows a typical flight envelope for a high performance military aircraft. Figure 15 is presented in terms of knots equivalent airspeed plotted as a function of Mach number. At sea level, maximum dynamic pressure occurs at about 1.2 Mach number. A typical bending-torsion flutter boundary is also shown in Fig. 15. The critical flutter speed is seen to occur in the transonic speed range between 0.95 and 1.1 Mach number. Above Mach 1.0, the flutter speed usually rises due to the aft shift in the center of pressure and the reduction in the lift curve slope. Consequently, usually above 1.2 Mach, flutter is not an issue. Note also that below 0.9 Mach flutter is usually not an issue. Thus, over a vast portion of the flight envelope, bending-torsion flutter is really not a concern. Unfortunately, the region where flutter is an issue is the region where our current production codes are the least accurate, Mach 0.9–1.2.

Figure 16 shows a typical high-performance military aircraft. A close look at the aircraft shows that a different external store is carried on each of the underwing pylon stations. Whereas this store load may not be totally representative of a typical mission, it does emphasize the multiple possibilities that can occur when external stores are carried. More discussion on this subject will be presented later.

A typical high-performance military aircraft will be designed to 7.33 G or higher load factor. The result of this is that the wing



Fig. 16 Typical high-performance military aircraft with external stores.

will be designed by strength considerations and wing stiffness will be a fall out. Basically, the wing will not be designed by stiffness considerations, and as a result, the clean wing (without external stores) flutter speed will exceed the required 15% flutter speed margin. Consequently, clean wing flutter should not be an issue for a high-performance military aircraft.

The wing control surfaces will usually be designed as irreversible surfaces without mass or aerodynamic balance. Coupled wing-control surface flutter is usually prevented by adding enough rotational stiffness in the actuator to raise the uncoupled rotation frequency above the second bending mode of the wing. Wing-control surface flutter can be an issue, however. In Ref. 39, a case of coupled aileron flutter on the T-46A aircraft is described. A significant outcome of this investigation was that the doublet-lattice method significantly overpredicts the effectiveness of the aerodynamic terms associated with trailing-edge control surfaces. A trailing-edge control surface mode with no aerodynamic balance will stiffen as dynamic pressure is increased. If the frequency of the rotation mode at zero speed is below wing bending, then at some speed the rotation mode will cross the bending mode and flutter may occur. Whether flutter occurs or not will depend on the dynamic pressure at the velocity where the modes cross. The result of this is that the aerodynamic terms, especially lift due to control surface rotation, associated with the trailing-edge control surfaces need to be reduced by as much as 50%.

Flutter problems with the vertical tail can be a design consideration for several reasons. First, coupled vertical tail/rudder flutter similar what was described for the wing must be addressed. Second, the loads that design the vertical tail will not necessarily lead to a strength-controlled design. Consequently, stiffness may be defined by bending-torsion flutter requirements. Third, because of stealth considerations, the rudder hinge line may end up being parallel to the trailing edge, and the rudder may be full span. These considerations may lead to a coupled rudder rotation bending-torsion flutter mechanism.

Of all of the surfaces on a high-performance military aircraft, the horizontal tail will usually be the most critical for flutter. In general, this surface will be all moving with restraint in the pitch-rotation mode being provided by the actuator. The flutter mechanism will be a coupling of the first bending mode with the pitch-rotation mode. In fact this flutter mechanism will usually size the stabilator actuator by defining the required stiffness. The challenge in the computation of the flutter speed for this mechanism comes from the free play in the pitch-rotation mode. The Joint Service Specification Guide allows up to 0.034 deg. However, geometry considerations usually make this very difficult to achieve, and typical values of 0.1 deg have been successfully demonstrated. The issue here is that the aerodynamic theory used must provide a way to address this nonlinearity. This usually requires the aerodynamic forces to be converted into the time domain using an appropriate transformation. An additional issue for an all-moving horizontal tail is that there is a discontinuity in the

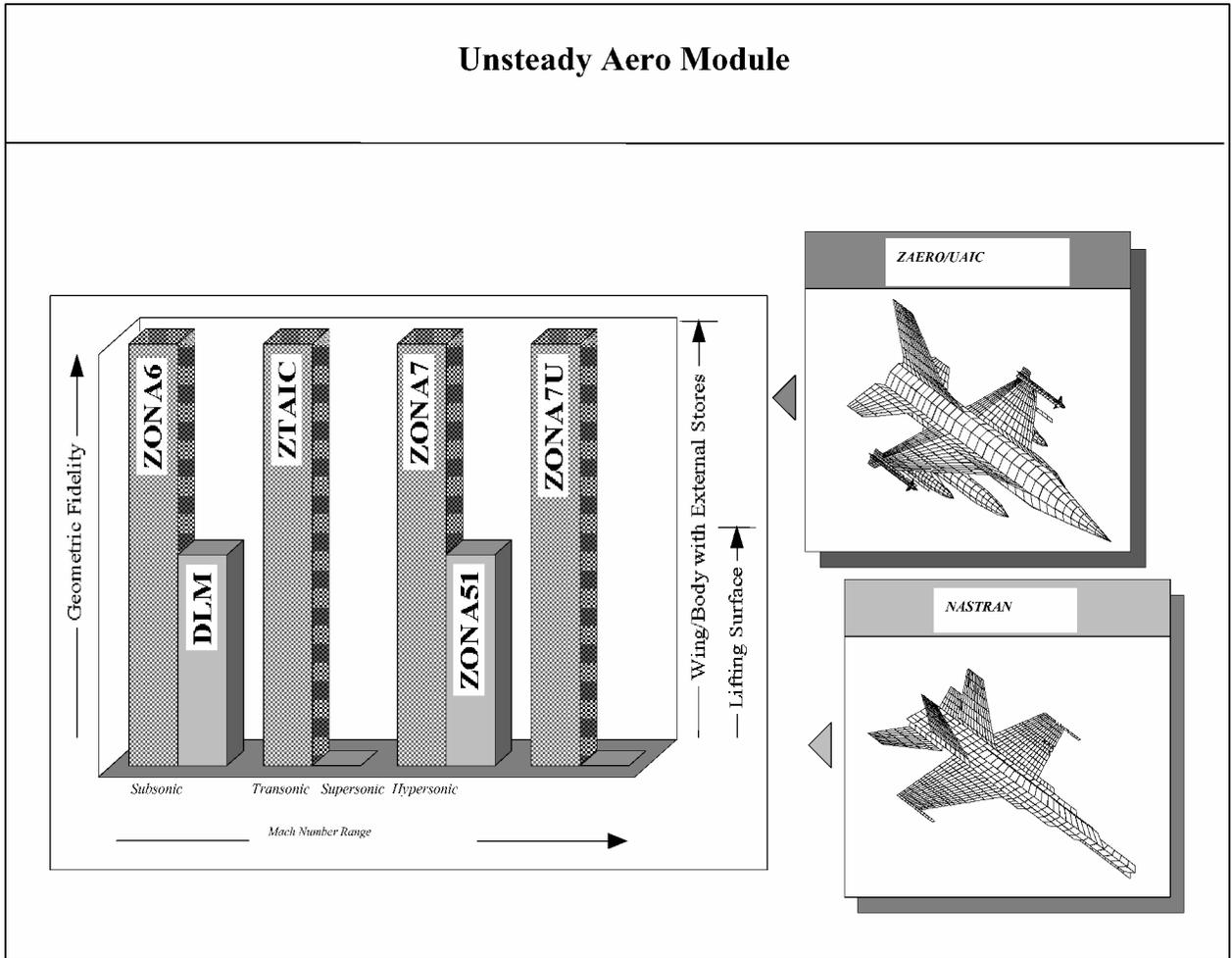


Fig. 17 ZONA code unified AIC approach vs NASTRAN aero module.

mode shapes at the root and the fuselage sidewall. This is usually not an issue in production codes such as doublet lattice that allow the mode shape to be broken up into different pieces such that slope continuity is not required.

It was stated earlier that clean wing flutter is usually not an issue for a high-performance military aircraft. However, it was also noted that these aircraft are required to carry external stores and that the number of possible combinations can be large. In fact, one estimate for the F/A-18E, which has three underwing pylons per side and a wing-tip missile, was that over 400,000 combinations of external stores are possible. All of these combinations must be cleared for flutter. This provides a requirement for whatever aerodynamic theory is to be used. The code must be very rapid. One way to accomplish this is through the calculation of aerodynamic influence coefficients. These can be extremely helpful, especially if it can be justified that aerodynamics on the external stores are not important. This assumption is usually adequate for screening purposes.

From this short discussion, four requirements for unsteady aerodynamic codes can be identified.

1) Accuracy is necessary in the transonic speed range. This speed range is likely to be where the critical flutter speeds will occur. If the basic code does not have the necessary accuracy, provisions for corrections must be available.

2) The code must be able to function in situations where the number of normal modes exceeds 100.

3) Corrections for nonlinear effects such as free play must be accommodated.

4) The codes must be fast. This is especially important where a large number of external stores must be analyzed for flutter clearances.

V. Modern High-Performance Unsteady Aerodynamics Code: ZONA

In Ref. 40, some of the recent advances made in the development of unsteady aerodynamics for flutter analyses are described. These codes, developed by ZONA Technology, are shown collectively in Fig. 17, where they are compared to the capability contained in the NASTRAN aeroelastic module. Several observations can be made from Fig. 17. First, a new subsonic code, ZONA6⁴¹ has been introduced as a substitute for doublet lattice. This code is based on the constant pressure panel method, and it has demonstrated improvements in modeling capability, especially for cases of high aspect ratio boxes that can result if high-reduced frequencies are needed. In general ZONA 6 is more robust in its modeling capability because of its ability to model arbitrary bodies.

Second, a new supersonic code, ZONA7 (Ref. 42) has been introduced as an improvement over the previously introduced ZONA51 that is contained in the NASTRAN aeroelastic package.

Third, a unified supersonic/hypersonic lifting surface method, ZONA7U,⁴³ that combines ZONA7 with piston theory has been introduced. Based on the framework of ZONA51, a unified lifting surface method, which improves on the concept of piston theory, has been developed that can account for wing thickness or incidence effects in supersonic and hypersonic flow. Cases computed by ZONA7U confirm that the effect of thickness is to reduce the supersonic flutter speed. This method extends the applicability of lifting surface methods to the Newtonian limit.

Fourth, a correction process that accounts for some of the nonlinear effects of the transonic speed range has been introduced. This procedure makes use of steady pressure data either computed from computational fluid dynamics (CFD) or measured from a

wind-tunnel test to compute the unsteady aerodynamic forces. Although this method accounts for shock location and strength, AICs are not calculated.

Finally, all of these codes have the ability to model bodies, and all of them produce AICs (except for the transonic method). These last two features address the requirements for speed, AIC-based codes, and modeling capability.

The modeling capability of the ZONA code is demonstrated in Fig. 18 for a generic fighter aircraft. The body has been idealized as a series of panels that are used to maintain the general cross-sectional shape of the body. For twin-engine high-performance fighter aircraft, the circular cross section implied by slender-body theory is usually not representative. When the body panels of the ZONA code are used, the aerodynamic surfaces can be located in their correct position in space, thus, maintaining the correct aerodynamic interaction.

One very important requirement for high-performance aircraft is that they carry external stores, and of course these configurations must be analyzed for flutter. In many cases, especially for screening purposes where the goal is to define the critical stores, aerodynamics on the stores are neglected. However, for the critical stores, those with the minimum flutter speed, or for those stores with large aerodynamic surfaces, an evaluation of the effects of store aerodynamics is required. Figure 19 shows a typical configuration. Note that in addition to the underwing stores, a detailed model of the tip missile has also been included. This store location is especially critical. Cases have been found where including aerodynamics on the aerodynamic surfaces of the tip missile has reduced the predicted flutter speeds by up to 100-kn estimated airspeed (KEAS). Fortunately, the modeling capability of the ZONA code can easily handle this situation. Note that the ZONA code can model these external stores for both supersonic and subsonic Mach numbers. In fact, other than the ZONA code, it is not known if there are any other unsteady aerocodes that can model the tip missile in the supersonic speed range for production flutter analyses.

The final feature of the ZONA code to be presented here is the code's ability to model nonlinear effects in the transonic speed range.

This feature will be demonstrated through its application to the F/A-18A limit-cycle oscillation that occurs when the aircraft carries certain external stores. During flight testing of the F/A-18A, a limit-cycle oscillation was found when heavy external stores are carried on the outboard pylon.⁴⁴ This limit cycle oscillation (LCO) is not predicted by conventional double-lattice techniques. What is predicted is a fairly abrupt flutter that if encountered in flight would hardly be expected to behave as a limit cycle. Figure 20 shows some typical flutter results obtained using linear AICs. As can be seen from Fig. 20, the damping in the unstable mode continues to grow as the dynamic pressure is increased. Although the results presented here were computed using ZONA6, similar results are obtained with doublet lattice.

Steady-state pressures for the transonic speed range were generated using the CFL3D CFD code, and these pressures were used with the ZTAIC procedure to generate modified generalized forces. Figure 21 shows results for the same flight condition as shown in Fig. 20 but obtained using the generalized forces modified by the steady pressures obtained from CFL3D. The damping computed for the unstable mode now shows that a small amount of structural damping could have a significant effect on the instability speed and,

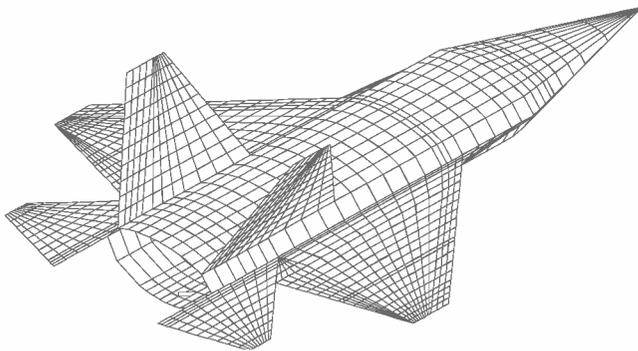


Fig. 18 Modeling capability for ZONA code demonstrated for generic fighter aircraft.

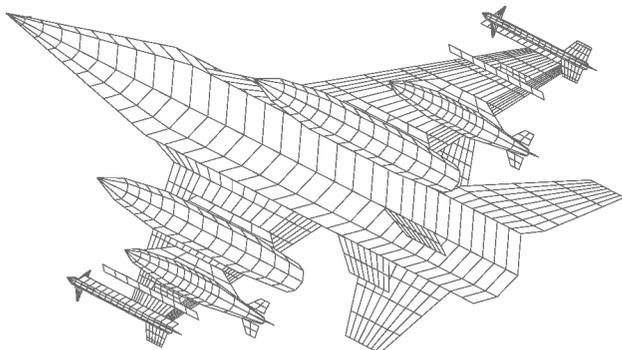


Fig. 19 ZONA code ability to model fighter-type aircraft with external stores.

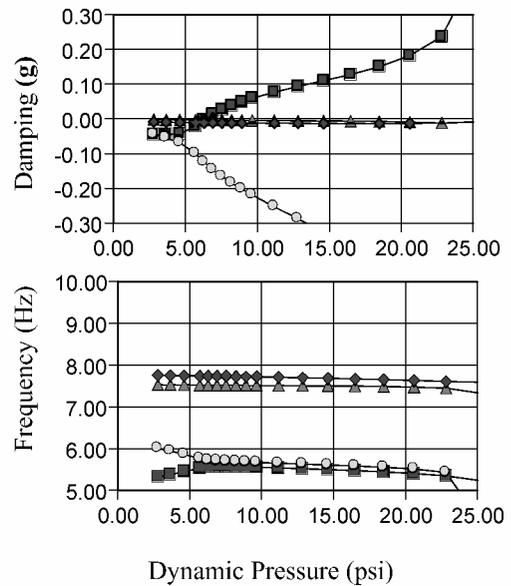


Fig. 20 Linear flutter results obtained using ZONA6 for Mach 0.9.

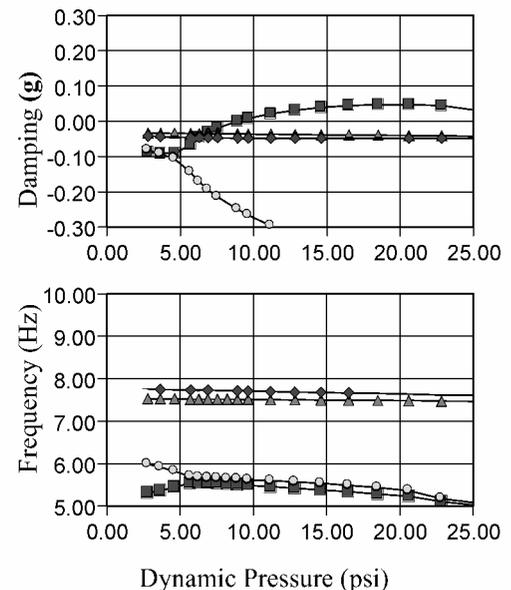


Fig. 21 Nonlinear flutter results obtained using ZTAIC.

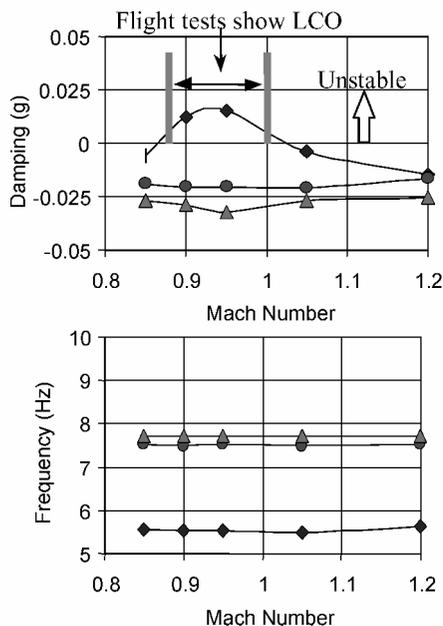


Fig. 22 Damping and frequency vs Mach number for LCO calculation based on ZTAIC: ♦, mode 5; ●, mode 7; and ▲, mode 8.

further, could lead to an LCO, which is what is obtained in flight. Whereas this is not a complete explanation of the LCO mechanism, it does show that modified linear methods may offer assistance in obtaining a solution.

Figure 22 shows a summary plot of the Mach range where LCO has been observed in flight. The instability as computed with the modified AICs is seen to be consistent with the speed range where the instability is observed in flight. In addition, the unstable damping never gets above a value of 0.02, which could be consistent with a lightly damped LCO.

In an earlier section, four features of a modern unsteady aerodynamics code were identified. In the preceding discussion, these features were shown to be addressed by the ZONA code.

VI. Future

The doublet-lattice method was developed over 30 years ago and today nothing has appeared that offers a significant improvement. It is anticipated that the doublet-lattice method will continue as the single most used unsteady aeromethod for a long time into the future.

The immediate need is for a code that can accurately predict flutter in the transonic speed range. Whereas CFD has made progress as a research tool, it has yet to demonstrate its value in a production environment. Corrections to flutter speeds computed using linear methods are still needed and are usually generated using transonic wind-tunnel flutter model testing. Reference 45 contains a good summary of the contributions made by the transonic dynamics tunnel at the NASA Langley Research Center over the past 30 years.

During a typical transonic flutter model test, between 5 and 10 measured flutter points are generated. This is usually enough to define the transonic dip for a specific configuration. To obtain this data, a wind-tunnel model must be designed, constructed, and tested, and the data must be analyzed. This process will require over a year's time. Even at present, existing CFD codes should be able to obtain five flutter solutions in one year. Thus, it is suggested that the first application for unsteady CFD codes could be as a replacement for transonic wind-tunnel flutter model testing. The results of a finite number of CFD solutions could be used as a replacement for wind-tunnel testing, assuming a validated code is available.

To evaluate the performance of existing CFD codes the U.S. Air Force, along with industry and university cooperation, under the Fixed Wing Vehicle Program, formulated the Aero-Structure-Controls Interaction Committee.⁴⁶ Two of the contracts sponsored under this activity were to evaluate existing CFD codes using real-world problems. Preliminary results from this activity are presented

in Ref. 47. One of the test cases included in the work reported in Ref. 47 is the F-15 stabilator snag leading edge that was described earlier and also reported in Ref. 35. The challenge is to predict using CFD the improvement in flutter speed observed in the transonic wind-tunnel flutter model tests but not predicted using doublet lattice. When this work and the other test cases are used as a base, resources could be devoted to developing an unsteady CFD code that could supply the next generation of aerodynamics tools for flutter analyses.

Rodden published a paper⁴⁸ over 25 years ago with a title similar to this paper. In Ref. 48, Rodden described the aspects of unsteady aerodynamics in several areas, and his comments on transonic flow are still pertinent to the discussion today. Three features were identified as being important to the transonic flow regime: airfoil thickness, incidence, and viscous effects. The recommendation that was made was that all three of these effects should be treated together or all of them should be neglected in achieving correlation with experimental data. Nothing has changed in the past 25 years that would invalidate this recommendation.

Although CFD may ultimately find a role in production flutter analyses, it has yet to make a significant contribution with possible exception of Ref. 49. In Ref. 49, a residual pitch oscillation (RPO) was analyzed using the NASA Langley Research Center Computational Aeroelasticity Program—Transonic Small Disturbance code.⁵⁰ Note that the three features identified in Ref. 48 were incorporated in this study, and the RPO was successfully analyzed. The role of validation in the development of unsteady CFD codes cannot be overlooked. A good source of data for validating unsteady aerodynamic codes is the RTO (formerly AGARD) technical report.⁵¹

VII. Conclusions

The objective of this paper was to provide an assessment of the state of the art in unsteady aerodynamics for production flutter analyses of high-performance military aircraft. This was done through a review of the history of unsteady aerodynamics codes over the past 30 years and through a review of the application of these codes to several high-performance military aircraft. It was shown that the doublet-lattice method, developed over 30 years ago is still the number one choice for subsonic analyses. For supersonic analyses, a number of codes have been used but none achieved the prominence that the doublet-lattice method has for subsonic analyses. However, the ZONA codes appear to have achieved significant recognition for this speed range.

The desirable features in any unsteady aerodynamics code were identified and finally an assessment of the future developments of CFD codes was presented.

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