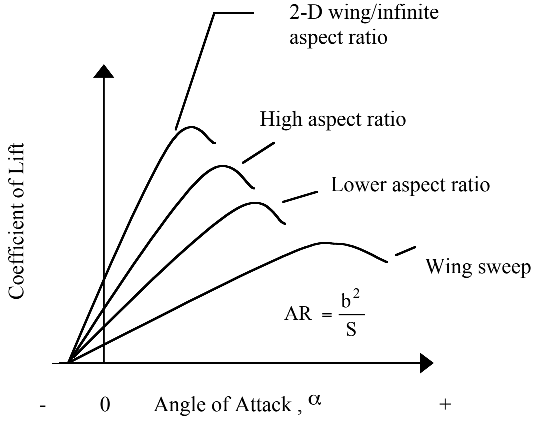
Flightlab Ground School
3. Three-Dimensional Aerodynamics

Real Wings

So far, we've been talking about two-dimensional airfoil sections. When you move from a two-dimensional, or "infinite," section to an actual three-dimensional wing of finite span, the slope of the lift curve changes. Figure 1 shows how decreasing the aspect ratio or increasing wing sweep decreases the C_L/α slope.

Figure 1
Lift Curve
Slope



Aspect ratio = wingspan (*b*) squared, divided by the wing area (*S*). Wingspan is measured directly from tip to tip.

Notice in the figure that, while the zero-lift angle of attack stays the same (the curves all start at the same point), the maximum C_L decreases and requires a higher angle of attack to attain.

Wing sweep tends to generate weaker adverse pressure gradients, which in turn causes the stall area of the curve to become flatter. Larger angle of attack changes are necessary to produce changes in lift, compared to wings of higher aspect ratio. Because lift changes more slowly on a swept wing, stalls are less pronounced than is usually the case in straight-wing aircraft that generate stronger adverse gradients—although drag increases quite fast with swept wings and the airplane can develop a high sink rate.

While watching the tufts on our trainers, you'll see that the boundary layer separation moves outward along the span, as well as up the chord. How that separation advances depends on how the local, section angle of attack varies along the span. This is key to understanding how a three-dimensional wing operates, but several concepts have to be brought together to make that understanding work.

Here's the short explanation: *A lifting wing produces a downwash in the air behind it, and an upwash ahead. We described earlier how a two-dimensional (no tips) wing section generates equal upwash and downwash. In the three-dimensional case, however, the downwash is greater because of the added wingtip vortices. Variations in downwash along the span behind the wing, caused by vortex effects, can produce spanwise variations in the oncoming flow ahead of the wing, and thus local differences in section angle of attack.*

As we'll see, that's also the reason why the slope of the lift curve changes with aspect ratio.

The pressure differences between the top and bottom of a wing, which generate the tip vortices, also produce an overall spanwise flow, as shown in Figure 2. Air over the top of the wing flows somewhat inward, while air on the bottom flows outward. The result is that small vortices form along the trailing edge where the inward and outward flows meet. Since the relative deflection of the two flows is smallest at the wing roots, the vortices there are less intense than at the tip. The weaker inboard trailing edge vortices quickly merge downstream with the stronger wingtip vortex.

Figure 3 suggests how the circulation generated by the tip vortex adds to the downwash behind the wing. The influence of the downwash actually extends ahead of the wing. The air ahead begins to be pulled down in response to the flow behind the wing, in proportion to the downwash velocity, even before the wing arrives. Remember the upwash ahead of the wing, from two-dimensional aerodynamics? The net result is a reduction in the upwash and a reduction in the effective angle of attack.

Figure 2
Trailing Edge
Vortex
Generation

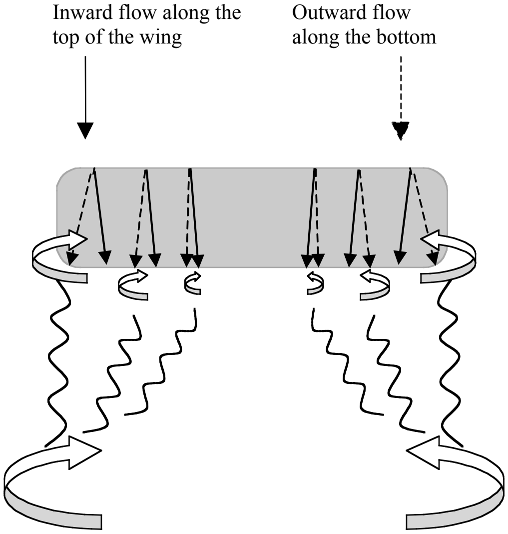
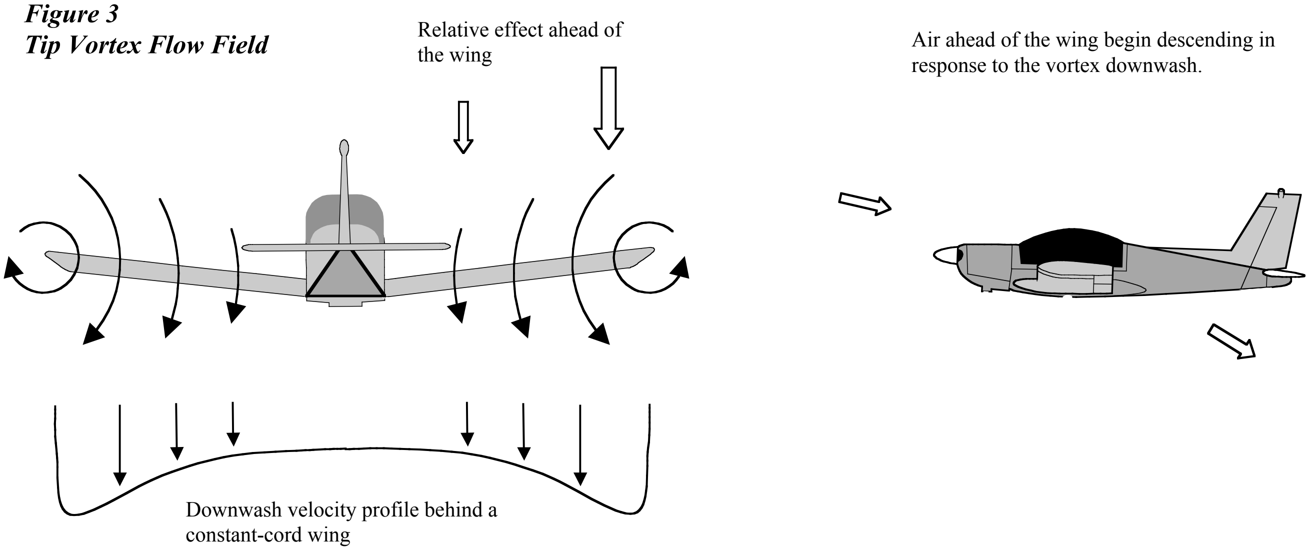


Figure 3
Tip Vortex Flow Field



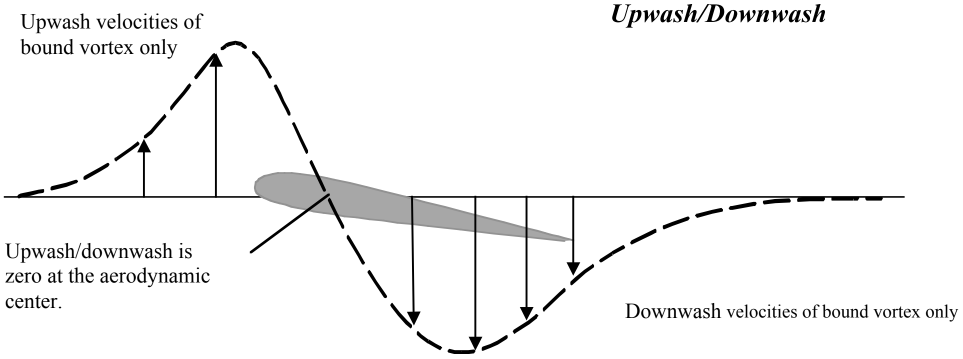


Figure 4
Bound Vortex
Upwash/Downwash

The next few pages may be difficult going, but if you survive you'll have some insight into wing stall patterns.

Figure 4 shows a common way of representing the components of the bound vortex around a given spanwise section of a finite wing, and Figure 5 shows what happens when the tip vortex is added to that circulation. Notice that the net upwash/downwash at the aerodynamic center is zero with the bound vortex alone. Adding the downwash from the wingtip vortex increases the vertical velocities of the total downwash behind the wing, and produces a *decrease in upwash velocities ahead of the wing, and a net downwash at the aerodynamic center*. (The aerodynamic center can be quickly defined as the location along the chord where changes in lift are considered to act. It's usually around 25 percent of the chord back from the leading edge. But that's an abbreviated definition.)

The wingtip and bound vortex together produce a final, vertical downwash velocity, *2w*, behind the section, that's twice the velocity, *w*, of the downwash at the aerodynamic center. Adding the freestream and downwash vectors together gives you the downwash angle, ε, as shown at the bottom of Figure 5

Remember that the Figures 4 and 5 show circulation around a *section* of a finite wing. At another section along the span the circulation could be different. The tip vortex may have greater or less influence due to distance or planform, or the wing might be built with a twist or a change in section profile.

Figure 5
Addition of
Tip Vortex

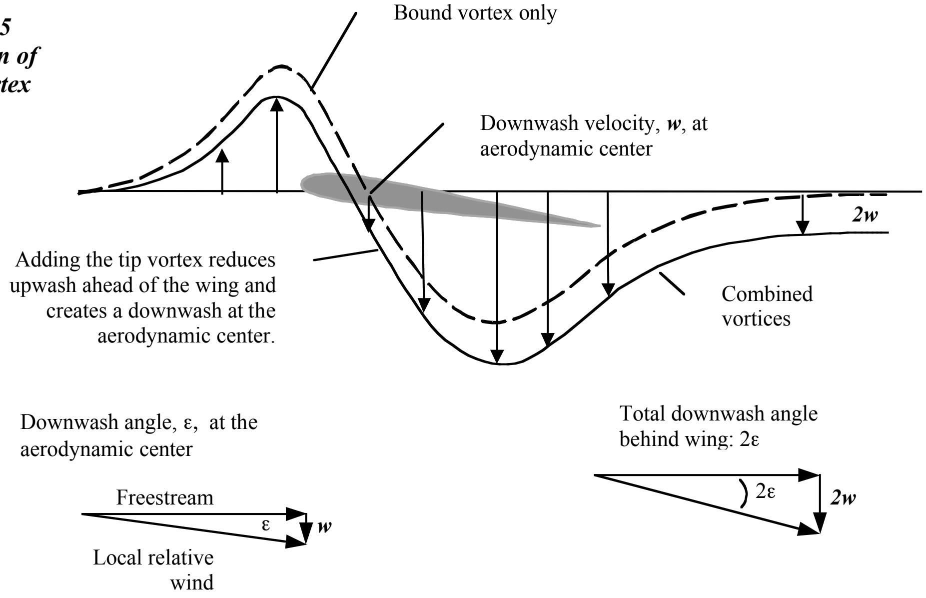
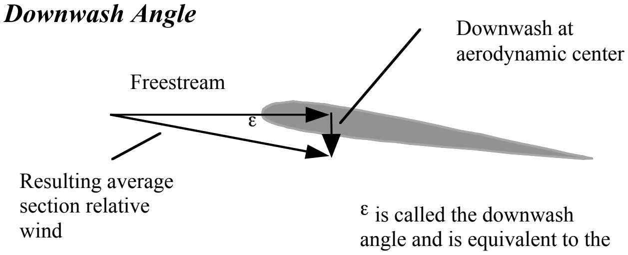
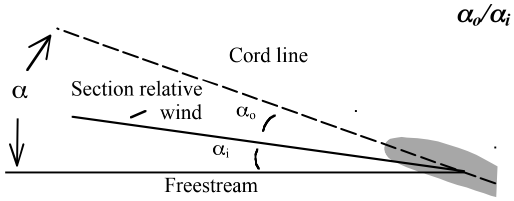


Figure 6
Downwash Angle



ε is called the downwash angle and is equivalent to the induced angle of attack, α_i.

Figure 7
α_o/α_i



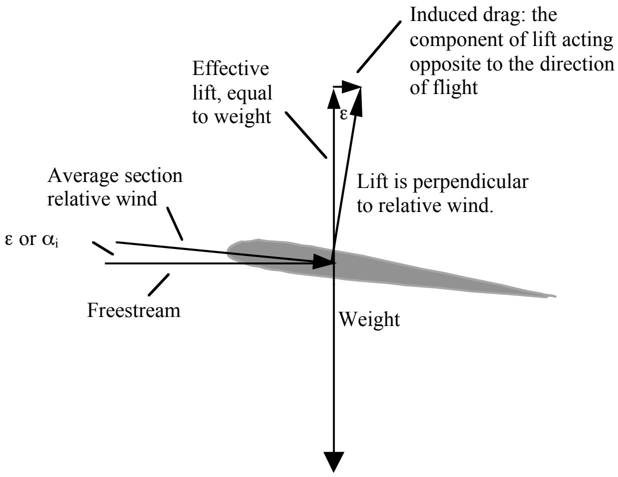
A wing's angle of attack, α, is the sum of induced angle of attack, α_i and section angle of attack, α_o. α_o generates lift; α_i tilts the lift vector and causes drag.

Figure 6 shows how the downwash behind the wing influences the effective angle of attack of a wing section. When you add the remote, freestream wind ahead of the section to the downwash at the section's aerodynamic center, the resulting vector is the section's average relative wind. This average relative wind, which is inclined to the freestream, is what the wing employs to create lift. Its inclination to the *freestream* is called the induced angle of attack, α_i. Its inclination to the wing *chord* is the local, average, section angle of attack, α_o. See Figure 7.

Once again, we can't talk about lift without talking about drag. *Because the lifting force is perpendicular to the local relative wind*, the inclination of the local relative wind, as shown in Figure 8, causes the lift vector to tilt back, opposite the direction of flight. The result is *induced* drag, as demonstrated in the figure by breaking the lift vector into horizontal and vertical components. Induced drag doesn't occur on a two-dimensional wing section: only on a real, three-dimensional wing with a tip vortex. Note that the angle ε between the freestream and the section relative wind is the same as the *induced angle of attack,* α_i, and also the same as the backward tilt of the lift vector.

The induced angle of attack, α_i, is directly proportional to coefficient of lift: double one and you double the other. But induced drag goes up as the square of the lift coefficient: doubling the C_L gets you four times as much drag.

Figure 8
Induced Drag



α, the wing angle of attack, is the angle between the cord line and the freestream relative wind.

With no wing twist, α tip = α root.

α_o , the section angle of attack, is the angle between the cord line and section relative wind. Because of vortex effects:

α_o tip < α_o root

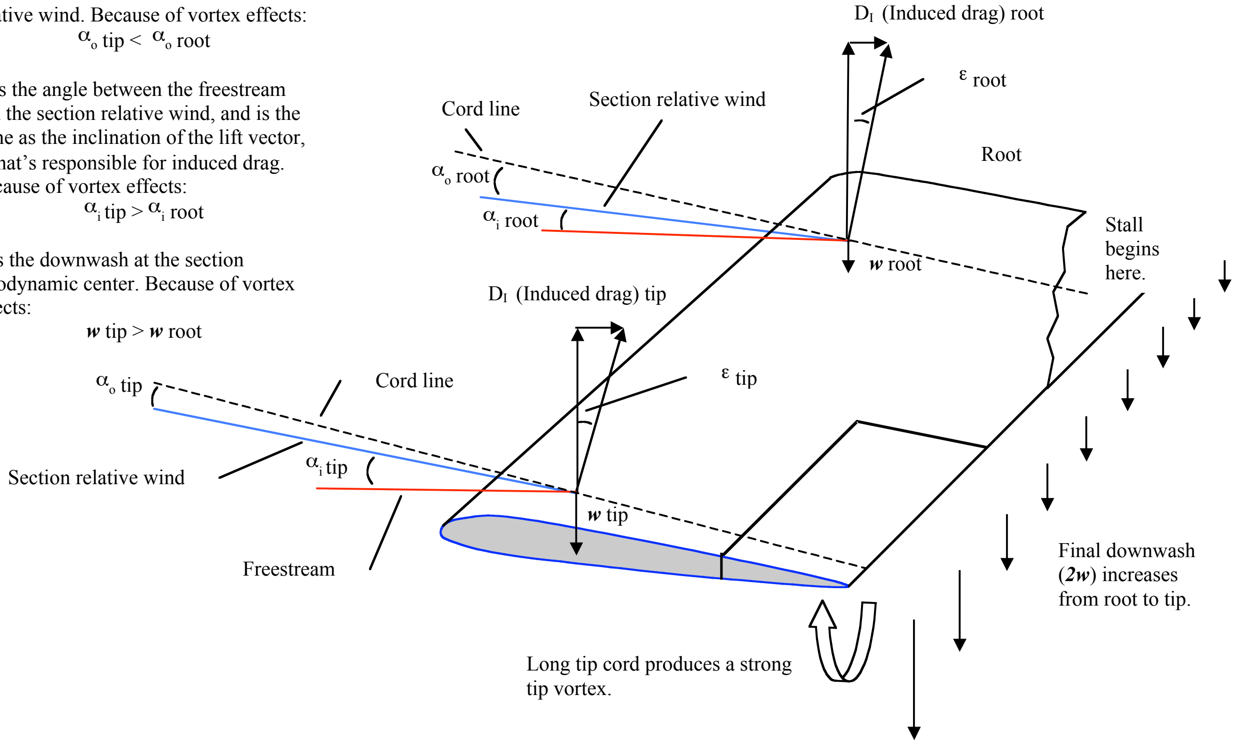
α_i is the angle between the freestream and the section relative wind, and is the same as the inclination of the lift vector, ε, that's responsible for induced drag. Because of vortex effects:

α_i tip > α_i root

w is the downwash at the section aerodynamic center. Because of vortex effects:

w tip > ***w*** root

Figure 9 Rectangular Wing Planform



At this point, you may have lost your patience keeping α, α_i, α_o, and ε straight. Don't worry; the important concept is simply that the downwash behind the wing affects the nature of the upwash ahead, and thus the local angles of attack along the span.

Figure 9 struggles to show that even when the angle of attack, α (angle between chord line and freestream), of the wing as a whole remains constant, its component angles α_i and α_o can be different at different spanwise section locations. That's because the downwash can vary along the span behind the wing, depending on planform effects and on the relative influence of the wingtip vortex. Wing sections operating at different local angles of attack, α_o, and thus *at different coefficients of lift*, can reach stalling angle of attack at different times.

A rectangular wing, like that of our trainers, produces a strong tip vortex and a total downwash that increases from root to tip (as Figure 9 illustrates). The greater downwash near the tip in turn reduces the outboard section angles of attack, α_o, relative to the inboard, for the reasons described above. Because the wing root operates at higher section angles of attack than the tip, the root is where the stall begins.

You'll observe this as you watch, and manipulate with the control stick, the spanwise movement of the tufts on the trainer's wing during stall entry. The tufts can't show you the downwash directly, of course, or section relative wind. But you will see the tufts responding to the accompanying changes in pressure pattern and to the expansion of the adverse gradient from trailing edge to leading edge and from root to tip, as section angles of attack increase and flow reversal and boundary layer separation start to occur.

Aspect Ratio

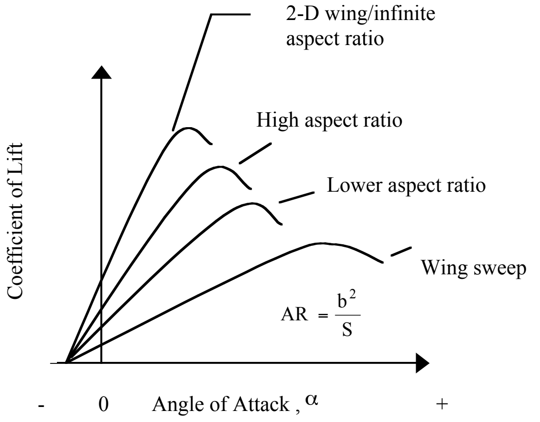
At the start, we mentioned the effect of aspect ratio, *AR*, on the slope of the lift curve. Here's some additional explanation. The formula for aspect ratio is:

$$AR = \text{wingspan}^2/\text{wing area}$$

Induced drag is inversely proportional to aspect ratio. Doubling the aspect ratio, for example, cuts induced drag in half. The smaller the aspect ratio (short wings) the faster induced drag will rise as we pull back on the stick and increase α and C_L. That's because as wingspan (and thus aspect ratio) decreases, the downwash from the wingtip vortex affects more of the total span.

Downwash distribution is the reason why low aspect ratio wings stall at higher overall angles of attack. Because the downwash from the tip vortex reduces the working, section angles of attack, α_o, over more of the wing, low aspect ratio wings need to operate at higher overall angles of attack, α, than longer wings to create equivalent lift. Therefore the slope of the lift curve in Figure 10 decreases with decreasing aspect ratio. Sweeping the wing also extends the influence of the vortex downwash inboard along the span, with similar effect.

Figure 10
Lift Curve Slope Revisited



Aspect ratio = wingspan (*b*) squared, divided by the wing area (*S*). Wingspan is measured directly from tip to tip.

Planform Differences and Stall Characteristics

Different wing planforms can produce significantly different downwash distributions behind the wing, and therefore different *lift distributions* along the span. You find the lift distribution by comparing section coefficients of lift, C_l, to the lift coefficient produced by the wing as a whole, C_L. The lift distribution determines the stall pattern.

When the downwash behind a constant-section, untwisted wing is uniform along the span, all sections of the wing will operate at the same section angle of attack, α_o, and section coefficient of lift, C_l. An elliptically shaped wing (like the one on your Spitfire) creates this sort of uniform downwash distribution.

Compared to a rectangular tip, the elliptical wingtip produces less total lift because of its reduced chord, and therefore a less intense tip vortex. The elliptical wing has a great advantage in generating the least induced drag compared to any other wing shape of the same aspect ratio. Since induced drag predominates at high C_L, the planform probably helped keep the Spitfire from losing energy in turns, where high g-loads require high lift coefficients. Elliptical wings are said to be difficult to build. As an alternative with nearly the same drag reduction characteristics, a tapered wing allows a compromise between drag and structural requirements.

Figure 11
C_l / C_L

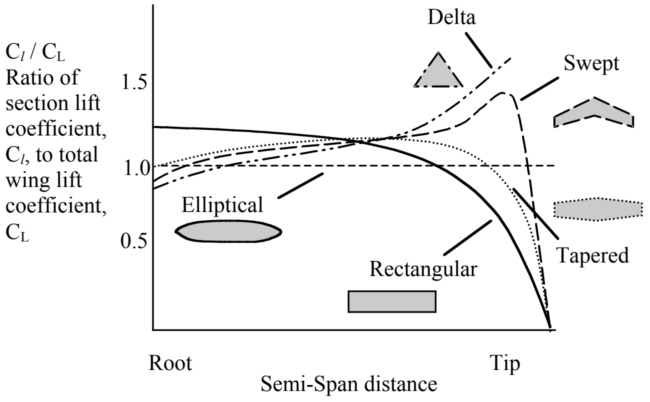
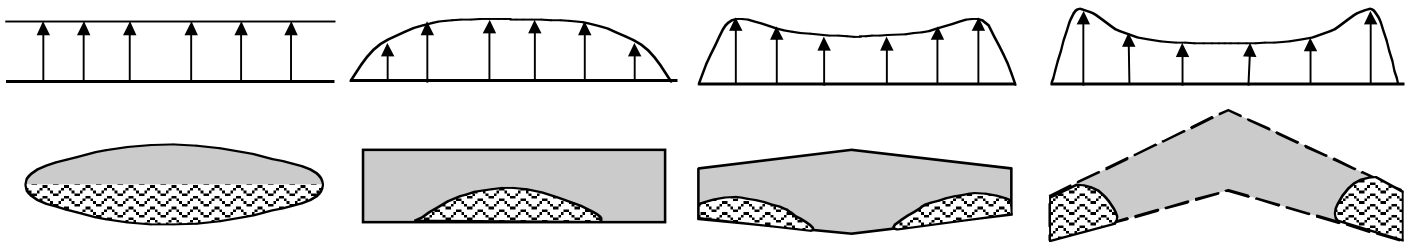


Figure 11 shows how the ratio between the section lift coefficient, C_l, and the coefficient of lift for the entire wing, C_L, varies between planforms. The elliptical wing has a constant ratio of 1.0. The lift distribution is uniform. As α increases, the sections all use up their lift potential at the same rate, and therefore will stall at about the same time. That can mean a sudden stall break, with little warning and ineffective ailerons.

The rectangular wing, however, starts with a C_l/C_L ratio higher than 1.0 at the root, where the section coefficients are greater than the wing

Figure 12
Stall Patterns
No Washout



coefficient as a whole. The ratio drops below 1.0 about two-thirds out, as the section coefficients become less than the wing's, and then goes to zero at the tip. As wing angle of attack, α, increases, the straight rectangular wing, with its skewed lift distribution, uses up its lift potential faster at the roots than at the tips. It stalls first at the roots.

The swept wing does just the opposite. It stalls at the tips because that's where it uses up its lift first. Figure 12 shows the relationship between stall pattern and the increase in local section C_l as α increases.

Although you pay a penalty in higher induced drag from the wingtips, the rectangular wing has optimal stall characteristics. The elliptical planform is the standard against which the efficiency of other wings is measured in terms of drag at subsonic airspeeds, but the rectangular wing sets the standard for behavior in stalls. Most of the gizmos that you find on other wings are designed to give them the benign stall characteristics and high-α lateral control more like that of a rectangular planform.

Stall warning can be better with a rectangular wing because the initial separation at the root can place the horizontal stabilizer in turbulent airflow, producing a warning buffet. This buffet is very evident in our training aircraft. When you see the wing root tufts start reversing you'll immediately feel the effect on the tail. In some aircraft the stick will shake against your hand as the elevator responds to the turbulence. That's the reversible control feedback that mechanical shakers are meant to simulate.

Roll control is naturally better with the rectangular wing as the stall approaches, because the ailerons work behind a lower section angle of attack and so remain in attached airflow longer into the stall entry. Geometric washout (twisting the leading edge of the wingtip down) and aerodynamic washout (changing the airfoil section toward the tip) are also used to adjust the lift distribution, keep the wingtips flying, and keep lateral control within bounds. Stall strips are used to adjust the spanwise stall pattern by tripping the root section into a stall at a lower angle of attack than would otherwise occur.

In out trainers, you'll be able to make a connection between the stall patterns that the tufts allow you to see and the resulting changes in aircraft lateral control.

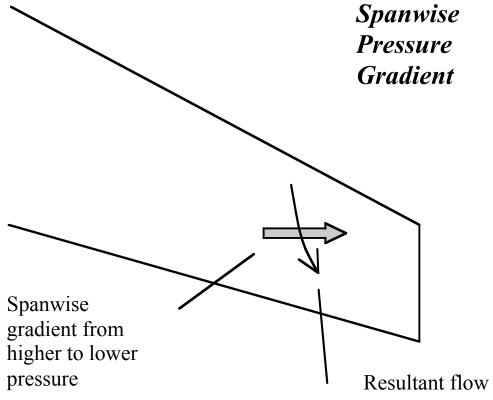
Swept-wing Characteristics

A tendency toward tip stall happens when you radically increase wing taper, sweep, or both, without also introducing a compensating wing twist or a change in airfoil section along the span. Taper or sweep shift the vortex and the downwash inboard, causing the tips to work at higher section coefficients of lift, α_o.

Aerodynamic stall warning can deteriorate seriously when the tips reach stalling angle of attack before the rest of the wing, if the turbulence produced by the stalling tips passes outside the span of the horizontal stabilizer, preventing a warning buffet.

Because the tip sections of a highly swept wing can operate at higher section angles of attack (thus at lower upper-surface static pressures) than the inboard sections, static pressure over the top of the wing decreases from root to tip. The resulting spanwise pressure gradient produces an outward, spanwise flow that intensifies with increasing wing angle of attack. The geometry of a swept wing encourages this tendency because inboard areas of higher pressure are directly adjacent to outboard areas of lower pressure, as shown in Figure 14. The spanwise flow tends to thicken the boundary layer toward the tips. The thicker boundary layer transfers less kinetic energy to the surface, and this lowers airflow resistance to the adverse pressure gradient along the wing chord, encouraging separation. On a swept wing the combination of higher tip section angles of attack, α_o, and a thicker, more easily separated boundary layer can cause the tips to tend to stall first. Stall fences along the wing chord, between the ailerons and wing root, were an early and often-seen solution.

Figure 14
Spanwise Pressure Gradient



Obviously, tip stall is bad for roll control, because of the airflow separation over the ailerons. And it's not good for pitch control, either. Due to the sweep angle, loss of lift at the tips will shift the center of lift forward. This shift causes a nose-up pitching moment, which can drive an airplane deeper into a stall or deeper into a high-g turn. The wing vortices also move inboard as the tips stall, thus increasing the downwash on the vertical stabilizer and the pitch-up tendency.

Figure 13
Surface Pressures