

Flightlab Ground School

2. Two-Dimensional Aerodynamics

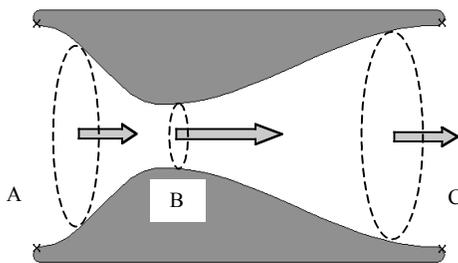
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For Training Purposes Only

Our Plan for Stall Demonstrations

We'll do a stall series at the beginning of our first flight, and tuft the trainer's wing with yarn before we go. The tufts show complex airflow and are truly fun to watch. You'll first see the tufts near the root trailing edge begin to wiggle and then actually reverse direction as the adverse pressure gradient grows and the boundary layer separates from the wing. The disturbance will work its way up the chord. You'll also see the movement of the tufts spread toward the wingtips. This spanwise movement can be modified in a number of ways, but depends primarily on planform (wing shape as seen from above). Spanwise characteristics have important implications for lateral control at high angles of attack, and thus for recovery from unusual attitudes entered from stalls. Our rectangular-planform trainers have excellent stall characteristics. Other planforms may need to be cajoled into behaving as if they were rectangular, stalling first at the root while the ailerons keep flying.

We'll first examine two-dimensional airfoil sections, then three-dimensional wing planforms. We'll only spend a few minutes during our flights watching the wing tufts, but those minutes can be full of information.

Figure 1
Velocity in a Venturi



Arrow length indicates velocity. Velocity actually starts rising before air enters the venturi.

$$(\rho AV)_A = (\rho AV)_B = (\rho AV)_C = \text{constant}$$

Remember Mass Flow?

Remember the illustration of the venturi from your student pilot days (like Figure 1)? The major idea is that the flow in the venturi increases in velocity as it passes through the narrows.

The Law of Conservation of Mass operates here: The mass you send into the venturi over a given unit of time has to equal the mass that comes out over the same time (mass can't be destroyed). This can only happen if the velocity increases when the cross section decreases. The velocity is in fact inversely proportional to the cross section area. So if you reduce the cross section area of the narrowest part of the venturi to half that of the opening, for example, the velocity must double at that point.

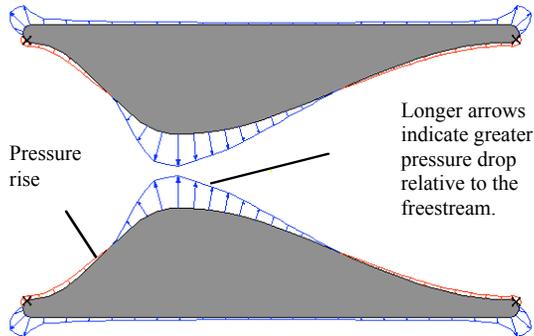
For a fluid (like air), the density, ρ , times the cross section area, A , of the venturi times the velocity, V , equals the *mass airflow*. Mass airflow, in and out, remains constant, so:

$$\rho AV = \text{Constant}$$

The above is known as the continuity equation.

Density (denoted by the Greek letter ρ , pronounced "roh") is the mass of air (in slugs) per volume (one cubic foot). Air is considered incompressible at the low, subsonic speeds we fly our trainers—so density doesn't change for us, only velocity.

Figure 2
Pressure Decreases as Velocity Increases



Remember Bernoulli?

Bernoulli's Theorem deals with conservation of energy. It tells us that for an ideal fluid (incompressible and frictionless) the total energy of the flow in the venturi remains constant. If we convert the total energy per unit volume of mass times flow rate into pressures, the sum of the static pressure and the dynamic pressure will equal a constant total pressure. Static pressure is the ambient pressure exerted by a column of fluid at a given level. Dynamic pressure is the pressure exerted by a mass of fluid in motion. In the formula below, static pressure is P_S . Dynamic pressure is $1/2 \rho V^2$, or one-half the density, ρ , of the fluid times its velocity, in feet-per-second, squared. P_T is the total pressure:

$$P_S + 1/2 \rho V^2 = \text{Constant } P_T$$

Or in English:

Static pressure + Dynamic pressure = Constant Total Pressure

So if velocity and thus dynamic pressure increases, static pressure will decrease. In the venturi in Figure 2, the increase in dynamic pressure with velocity produces a decrease in static pressure as the tube narrows. The static pressure then rises again as the tube widens downstream and the airflow slows down.

Of course, air isn't an ideal fluid: It's compressible and viscous. Compressibility

obviously becomes important approaching the speed of sound, but for present purposes can be ignored. But we won't ignore viscosity for long, because the nature of the airflow within the boundary layer over a wing depends on friction.

After looking at the behavior of velocity and pressure in a venturi, it's tempting to declare that the reason airflow accelerates over the top of a wing, and static pressure consequently decreases, is that a wing is just one half of a venturi, with the mass of the atmosphere playing a roll equivalent to the other half. That's a valid way of thinking about it, and easy to visualize. But aerodynamics is a subject in which alternative visualizations exist side-by-side. A different but not necessarily contradictory understanding of the acceleration of flow has to do with the idea of *circulation*. Circulation in turn gives us a way of visualizing the generation of wingtip vortices, and understanding how the strength of the vortical flow relates to the particular conditions under which the wing was producing lift. We'll return to this farther on.

Two-Dimensional Aerodynamics

Streamlines

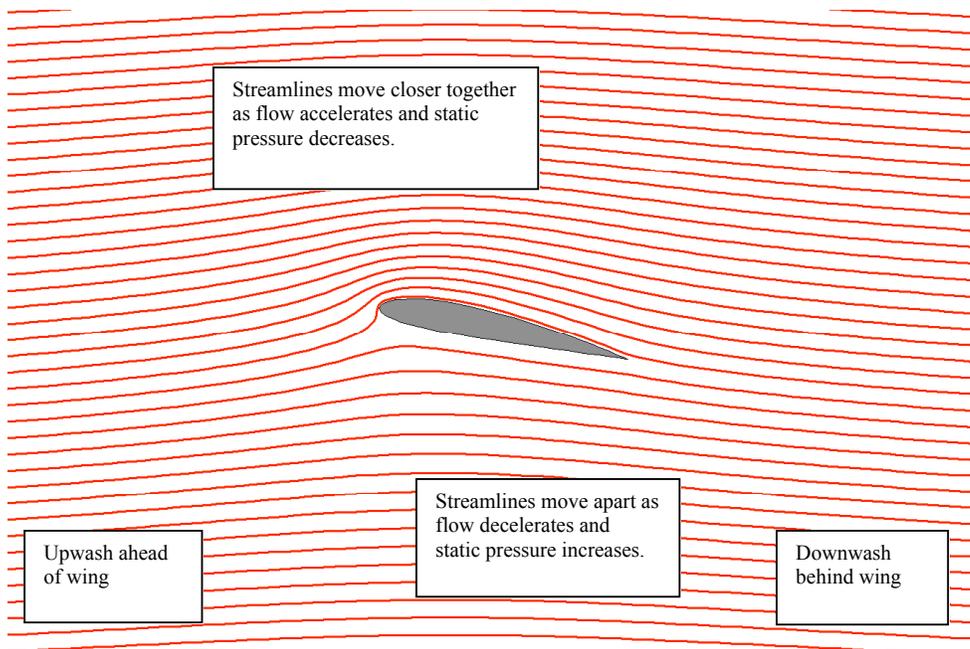
An airfoil section is equivalent to a hypothetical wing of infinite span (therefore no tips) or to a wing model in a wind tunnel, when the model extends right to the tunnel walls. Because of the absence of the spanwise flow induced by the presence of wingtips, no tip vortex and no variations in downwash behind the wing occur. A two-dimensional representation is all you need to depict what's happening.

The streamlines generated by injecting smoke into a wind tunnel, or calculated in a computer simulation (Figure 3), allow us to visualize not just the direction of flow around a wing, but also its velocity and pressure. The flow direction at any instant or point along a streamline is always tangential to the line. An important feature of streamlines is that air particles never cross them; adjacent pairs of streamlines thus behave like the walls of a flexible tube. When the flow accelerates, the resulting decrease in static pressure within them causes the streamtubes to contract, and the streamlines move closer together. The distance between streamlines is thus an indication of relative velocity and static

pressure. Notice in the figure how the streamtubes passing over the leading edge contract, indicating an accelerating flow and a pressure decrease. As they move down the wing they expand, indicating a decelerating airflow and a pressure rise. This is shown in closer detail in Figure 5.

Note the upwash in the airflow ahead of the wing, and the downwash behind. In the two-dimensional case illustrated, the upwash and downwash angles are equal. In the three-dimensional case of a finite wing, the tip vortex can add significantly to the downwash behind the wing, as we'll see later.

Figure 3
Streamlines
Upwash/Downwash

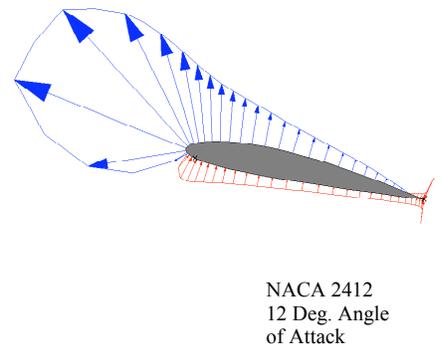
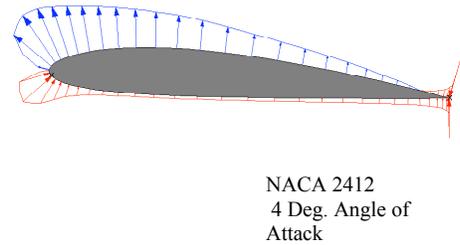
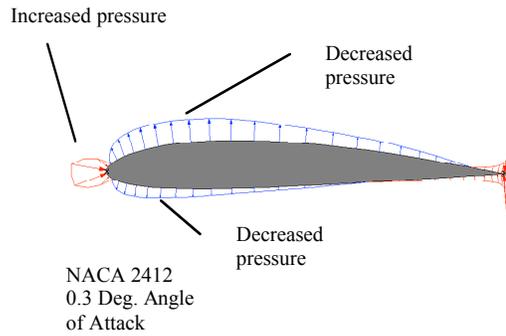


Pressure Distribution

Figure 4 shows the surface pressure distribution along an airfoil section at three different angles of attack. The longer arrows represent increasingly higher or lower static pressures, as indicated, relative to the static pressure of the freestream, undisturbed air ahead of the section. The velocity over the forward, upper part of the wing increases as the angle of attack increases, resulting in a greater decrease in local static pressure, as well as a shifting of the pressure pattern. The illustrations show how the point of lowest static pressure (longest arrow) moves forward as angle of attack, α , increases.

Notice that at low α (as in the top illustration) the static pressure can drop below freestream under the wing as well as above. Lift results as long as the overall reduction in pressure above the wing is greater.

Figure 4
Pressure Distribution
versus Angle of Attack

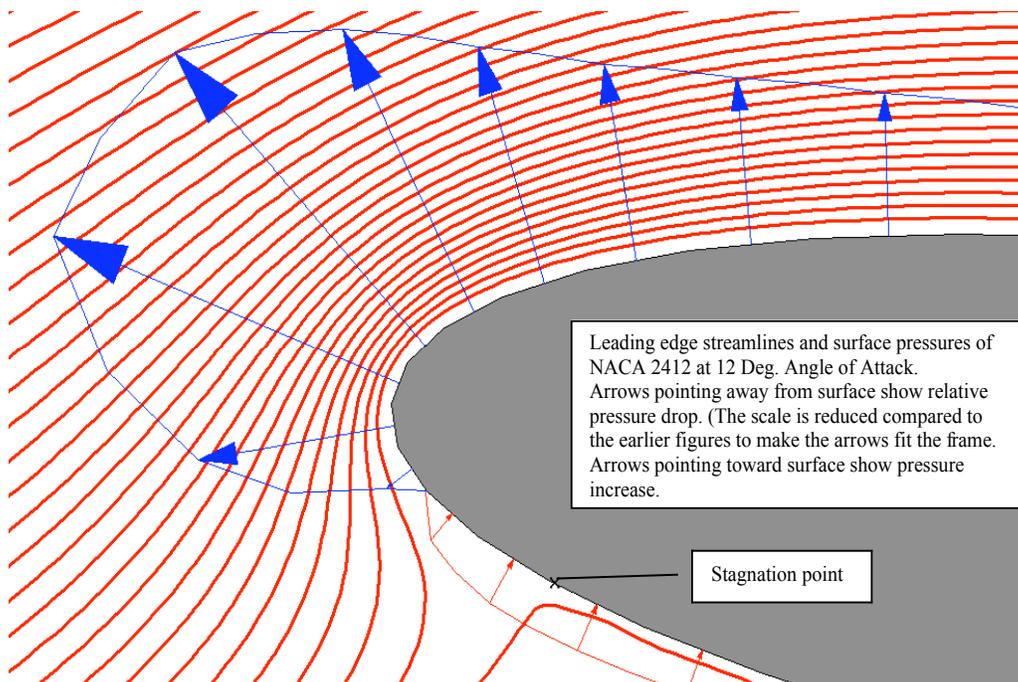


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Figure 5 illustrates streamlines and surface pressures together. Pressure is highest at the stagnation point (where it equals static plus dynamic pressure). Here airflow comes to a stop, and the streamlines split on either side to follow either the upper or lower surface. (Stagnation pressure is what the pitot tube senses. The static side of the pitot system eliminates the static component, and the remaining dynamic pressure appears as an airspeed indication.)

The low pressure generated by the acceleration of the flow over the leading edge creates a suction that draws the airflow forward from the stagnation point—against the general, leading-to-trailing-edge flow. Below stall, any increase in α further accelerates the upper-surface flow and further decreases the pressure around the leading edge. Because the increasing suction pulls additional lower-surface air forward, the stagnation point actually moves aft along the lower surface as α increases. It also moves aft when you extend the flaps or deflect an aileron down.

*Figure 5
Streamlines
and Surface
Pressures*



Pressure Gradient

Air flowing over the top of the wing initially moves through an area of decreasing static pressure (Figure 6). Since higher pressures flow toward lower, this *favorable pressure gradient* encourages the airflow. But once past the point of highest velocity and lowest static pressure on the airfoil, local static pressures begin to rise (although still remain negative) and the pressure distribution tends to retard the flow. This *adverse pressure gradient* becomes steeper and occupies more of the airfoil as the angle of attack, α , increases and the low-pressure over the wing intensifies and shifts forward (Figure 7).

Figure 8 defines the concept of coefficient of pressure, C_p , and shows how its distribution changes along the chord as α rises. Negative values mean local static pressures lower than freestream static pressure; positive values mean local static pressures higher than freestream static. Note how the adverse gradient increases over the top of the wing when α rises, as indicated by the increasing negative slope.

As α rises, the adverse gradient increasingly retards the airflow within the boundary layer, until the boundary layer ultimately separates from the surface, like a sheet blown away from beneath (Figure 9). As adverse pressure rises the separation point moves forward; lift drops and the airfoil stalls.

Figure 6
Pressure Gradient

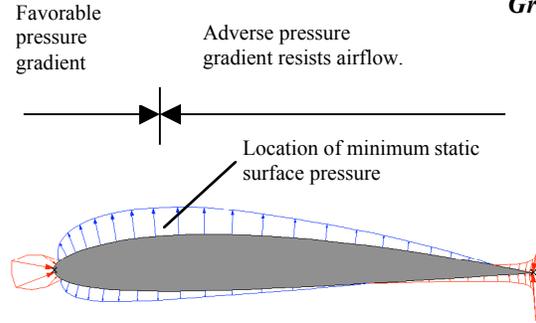


Figure 7
Increase in Adverse Pressure with Angle of Attack

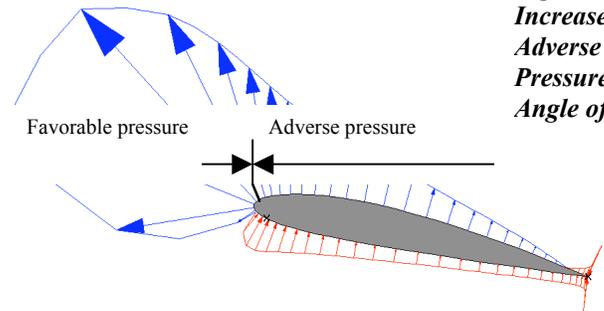


Figure 9
Separation

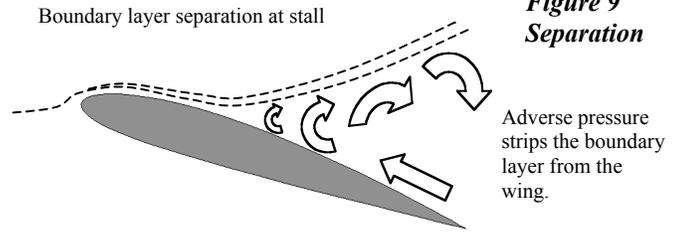
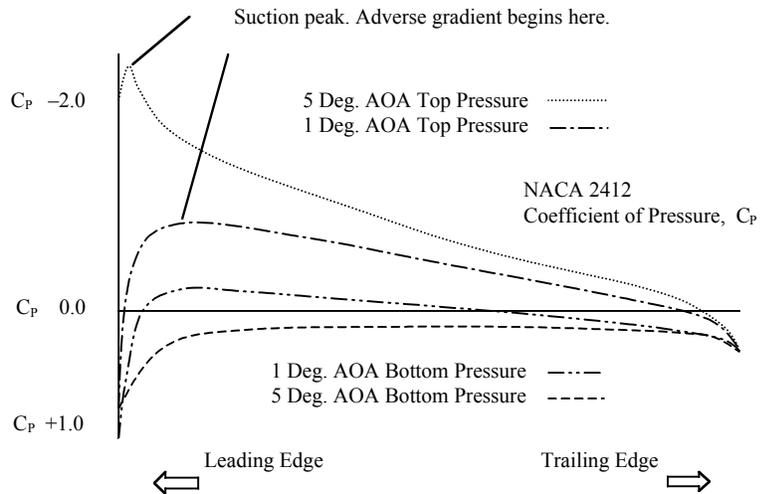


Figure 8
Coefficient of Pressure, C_p



Two-Dimensional Aerodynamics

Boundary Layer

In exaggerated scale, Figure 10 shows the regions of the boundary layer atop the wing, and the changing velocity profile within the boundary layer, as indicated by the length of the arrows. In both the laminar and turbulent regions, the viscosity of air causes the particles right next to the wing to come to a halt, due to friction. Their velocity relative to the wing is zero. The particles flowing immediately above are slowed down almost to zero by friction, and they in turn slow the particles above them. Initially, a profile of shear layers (lamina) develops, with the velocity of the layers increasing with their distance from the surface as the effect of friction diminishes. By definition, the boundary layer is the area from the surface out to the point where the flow reaches 99 percent of the freestream velocity.

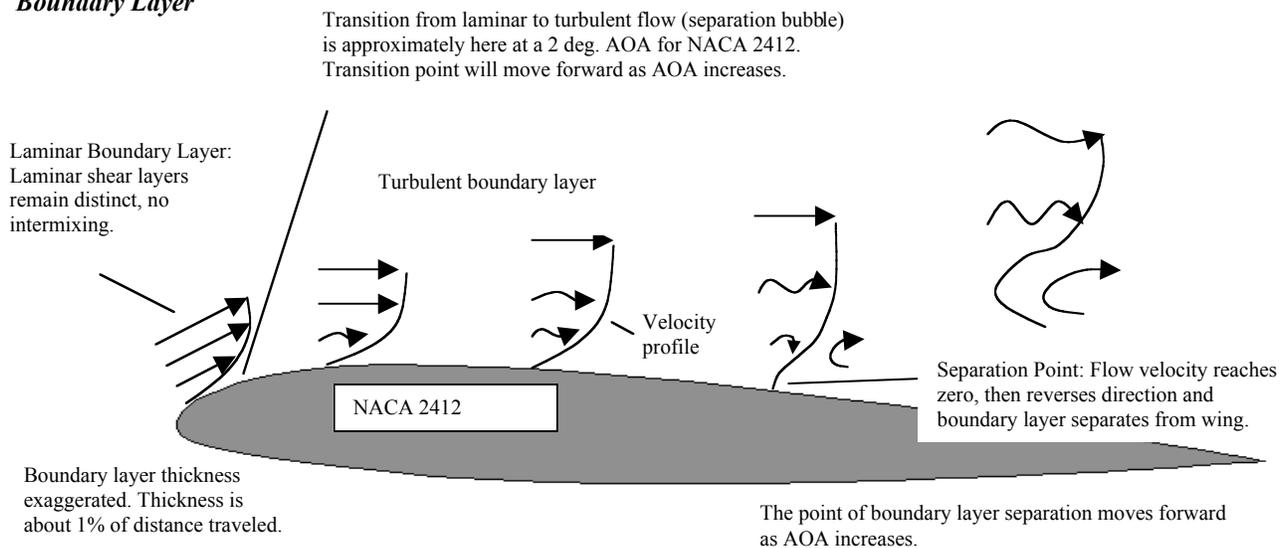
The viscous, frictional forces generated within the boundary layer are responsible for the component of drag known as *skin friction* drag. Outside of the boundary layer, viscosity has no important effect when it comes to predicting airfoil characteristics.

The initial, laminar region of the boundary layer is very thin. The flow remains layered—there's no interchange of fluid particles across the lamina. Moving downstream from the leading

edge, the laminar flow gains kinetic energy as it accelerates through the favorable pressure gradient. The favorable gradient also helps damp out irregularities in the flow. But the flow begins to slow as it enters the destabilizing resistance of the adverse gradient. The laminar boundary layer separates from the wing, becomes turbulent, and then reattaches, forming a *separation bubble* as it makes the transition. Roughness on the wing surface can cause the boundary layer to trip prematurely from laminar to turbulent flow.

Whether the laminar flow reattaches as turbulent flow depends on Reynolds number—on the ratio of the inertial forces to the viscous forces within the flow. Low Reynolds numbers are associated with laminar flow (viscous forces prevail), high Reynolds numbers with turbulent flow (viscous forces unimportant, inertial forces dominant). If the Reynolds number is too low, a detached laminar flow won't reattach as turbulent flow. Up to a point, an airfoil produces higher maximum lift at higher Reynolds numbers, because the reattached turbulent flow can better resist the adverse gradient and remain attached to the wing at higher angles of attack. Wind tunnel data for airfoil sections often includes curves for different Reynolds numbers and surface textures. (Reynolds number is more complicated, but the above gives you a start if the concept is new.)

Figure 10
Boundary Layer



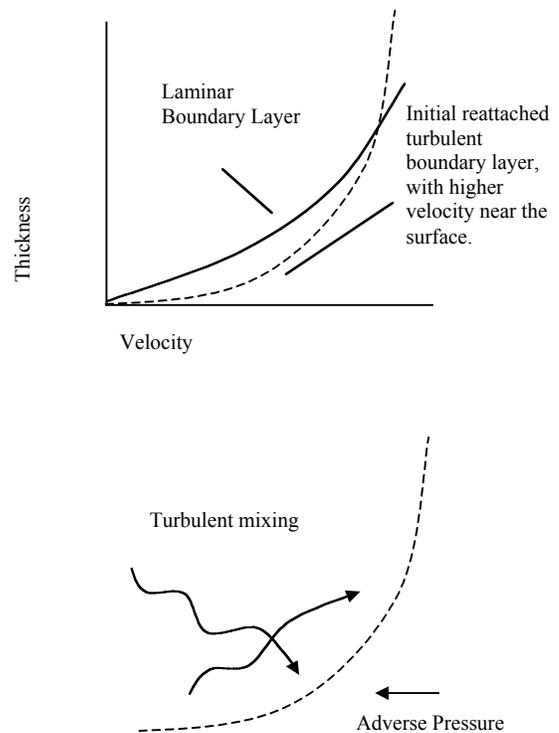
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In Figure 11, you can see that the velocity gradient curve in the initial turbulent region (dashed line) starts out shallower than in the laminar region, but as the curve rises becomes steeper and the boundary layer grows thicker. The turbulent boundary layer produces a lot more drag than the laminar layer (laminar-flow wings achieve their low drag by moving the lowest pressure point farther back along the chord, thus extending the laminar region). On the other hand, a turbulent layer stays attached to the wing better than a laminar layer as angle of attack rises, because the turbulence transfers kinetic energy down to the surface, which helps overcome the adverse pressure gradient. (This turbulent energy boost is the principle behind vortex generators. The vortices add energy to the flow, delaying separation caused by adverse pressure.) Figure 10 shows how the velocity profile of the turbulent boundary layer changes along the wing chord as the retarding effects of the adverse gradient accumulate.

The tufts on the trainers can give you an idea of how the turbulent boundary layer increases in thickness. The tufts themselves would trip any laminar to turbulent flow. At low angle of attack the trailing tips of the first row of tufts should lie fairly quietly against the surface of the wing, but by the second row the flow will have become more turbulent and the tips will shake perceptibly. The tips will show increasing movement, bouncing between the wing and the top of the boundary layer, as you look farther back along the chord. You can easily see how the boundary layer thickens as it goes downstream.

More important, the tufts show how flow reversal and turbulent boundary layer separation move up the chord from the trailing edge. You'll be able to see the effect of the adverse pressure gradient intensify as angle of attack rises. The turbulent boundary layer's separation point will move forward along the chord and the tufts will reverse direction, the free ends actually pointing toward the leading edge. The reversal of the tufts in order back up the chord means that the wing is generating a larger and larger turbulent wake. Pressure drag is rising rapidly.

Figure 11
Velocity Profiles



Intermixing in the turbulent boundary area brings high-kinetic-energy (high inertia) particles down toward the surface, increasing the boundary layer's ability to overcome the adverse pressure gradient. Intermixing also sends low-energy (low inertia) particles up to the top, delaying the return to freestream velocity and causing the boundary layer to become thicker.

The higher velocities closer to the surface increase friction in the turbulent boundary layer, and thus friction drag.

Two-Dimensional Aerodynamics

The Lift Curve

Figures 12 and 13 show typical lift curves, which plot angle of attack against coefficient of lift (C_L). You may have grown accustomed to using the term coefficient of lift (or just as happy not using it) without quite remembering how it's derived. It's just an indication of how efficiently an airfoil shape turns dynamic pressure (defined earlier as one-half the density of the air times velocity squared: $1/2\rho V^2$) into lift at any given angle of attack. S stands for total wing area in square feet. L stands for lift in pounds.

$$C_L = \frac{L}{1/2\rho V^2 S}$$

Looking at the formula, it's clear that the more lift generated for a given combination of dynamic pressure and wing area, the greater the C_L . The formula for dynamic pressure is typically shortened to q , so that the above becomes:

$$C_L = \frac{L}{qS}$$

You can determine q in the cockpit, in pounds per square foot, simply by multiplying the square of the indicated airspeed in miles per hour by 0.0025577. That's q . Then multiply the result by the wing area, found in the aircraft manual, and divide the product into the aircraft weight (since lift equals weight in equilibrium flight). The result is your current C_L for the wing *as a whole* (the coefficient at a given *section* along the span, called C_l to make the distinction when necessary, is often different than that of the wing as a whole). At a constant IAS, your C_L must slowly decrease as you shed fuel weight and need less lift. Any change in airspeed (thus in q) requires a change in C_L for level flight.

It's good aerodynamics, and good piloting technique, never to think about lift without also considering its inevitable pal, drag. Substituting drag for lift, we derive the coefficient of drag, C_D , just as above. D stands for drag in pounds:

$$C_D = \frac{D}{qS}$$

In a lift curve, as in Figure 11, the coefficient of lift initially shows a linear increase with angle of attack. A slope of about 0.1 in lift coefficient increase for each degree increase in angle is

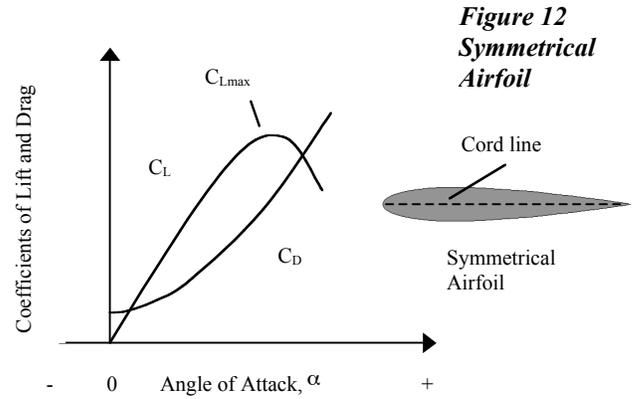


Figure 12
Symmetrical
Airfoil

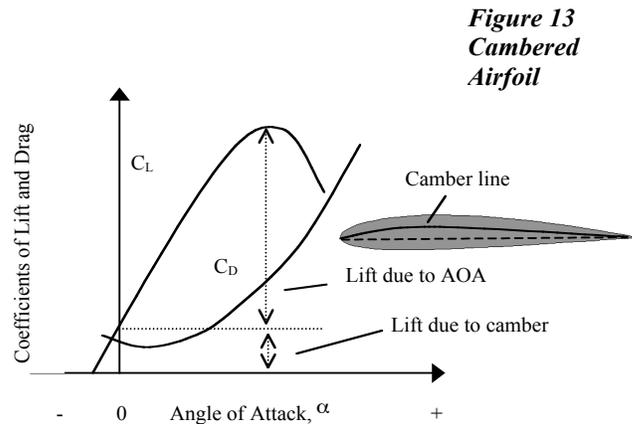


Figure 13
Cambered
Airfoil

typical for all two-dimensional airfoil section curves. Variations in slope come from planform, as we'll see.

The curve begins to shallow and reverse as airflow separation occurs on the upper surface of the airfoil. The point where maximum C_L versus α is reached ($C_{L,max}$) marks the stalling angle of attack.

At $C_{L,max}$, depending on airfoil shape, airflow separation may have already reached some 20 to 50 percent of the chord. Notice that an airfoil will still produce lift, *and a lot more drag*, even past the stalling angle of attack.

The airfoil in Figure 12 is symmetrical, and characteristically produces no net lift (and its minimum drag) at zero angle of attack. A cambered airfoil, as in Figure 13, can produce lift at small negative angles of attack. Its lift curve shifts up and to the left (producing a higher $C_{L,max}$), compared to a symmetrical airfoil's. The drag curve shifts to the right.

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Note how the rates of change in the lift and drag curves vary with angle of attack. The lift curve shows a constant rate until near its peak, when the slope diminishes so that a given change in angle of attack produces a smaller change in lift. The drag curve is different. At low angles of attack, drag doesn't change much, but as the angle of attack approaches stall, drag increases at an accelerating rate. You learn this, at least implicitly, when you learn to land an airplane. The technique of balancing the rates of change of lift and drag at high angles of attack is one of aviation's foundation skills. The technique can depend on whether the wing is long or short, swept or straight, as we'll note.

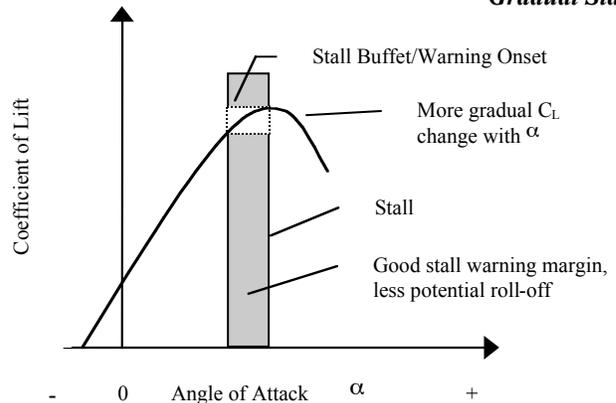
Airfoil thickness, the amount of camber and position of maximum camber along the chord, the radius of the leading edge, plus Reynolds and Mach numbers, are all factors that affect the velocities and pressures and therefore the aerodynamic forces generated by an airfoil. They also affect the characteristics of the boundary layer, its profile and turbulent transition, and its separation from the wing at high angles of attack.

Increasing the camber of an airfoil increases its $C_{L,max}$. It also tends to increase the adverse pressure gradient as angle of attack rises, which in turn encourages earlier boundary layer separation, making the section then stall at a lower angle of attack, relative to one with less camber. A wing in which camber has been increased by lowering the flaps will stall at a lower angle of attack (See Figure 17).

It's usually best if the separation of the turbulent boundary layer from the wing surface moves slowly forward along the chord as angle of attack rises near the stall. This allows a relatively gradual, parabolic change in the slope of the lift curve as the section approaches $C_{L,max}$, and results in less abrupt stall characteristics, better opportunity for aerodynamic stall warning (which also depends on wing planform and tail design), and less roll-off tendency if one wing starts to stall before the other. (See Figures 14 and 15.)

But abrupt turbulent boundary layer separation is sometimes useful. Modern, high-performance aerobatic wings often have large leading-edge radii and then become flat from the point of maximum thickness back to the trailing edge. The profile of these "ice cream cone" wings places the location of maximum thickness

Figure 14
Gradual Stall



forward on the chord, which limits the possible region of favorable pressure gradient and causes early laminar-to-turbulent boundary layer transition. Compared to wings designed for better laminar flow, they tend toward higher drag. Drag helps speed control in vertical down lines, but the major aerobatic benefit is the tendency for the stall separation point to remain near the trailing edge as angle of attack increases, and then suddenly to move forward up the chord. This allows the wing to hold airflow attached during high-g maneuvers, but stall abruptly for quick snap roll, spin, and tumbling entries. Wing tufts show how rapidly the separation point advances and how quickly the stall breaks, as you'll see in our ground-school video.

Leading-Edge Stall

A thin airfoil with a sharp leading edge radius can suffer sudden leading-edge stall (or simultaneous leading-edge and trailing-edge stall) due to the sudden bursting of the leading-edge separation bubble. The bubble occurs where the laminar flow separates from the wing and reattaches as turbulent flow. As angle of attack rises, the bubble follows the suction peak (Figure 8) forward. As the curvature of the wing increases, the detached laminar flow can no longer “make the turn” necessary to reattach as turbulent flow. The bubble bursts, causing rapid, complete boundary layer separation along the entire chord. This sharpens the peak of the lift curve, with the result that, near stall, a small change in angle of attack produces a sudden drop in C_L . This can lead to an abrupt stall with limited or no aerodynamic warning, and to a sudden roll-off when inevitable variations in wing surface or contour cause the separation bubble to burst on one wing ahead of the other.

Depending on how the airfoil section varies along the span, it’s possible for one part of a wing to have a trailing-edge stall, while another part has a leading-edge stall, or demonstrates a combined leading-edge/trailing-edge stall.

Learjet wings built before the introduction of the Century III wing section had roll-off problems due to asymmetrically bursting separation bubbles. A stick pusher was necessary to keep the wing out of stall territory. In addition to adding inboard stall strips and stall fences, improving the pusher-off stall characteristics involved breaking the bubble into stable, spanwise segments by mechanically tripping the laminar flow with small triangular shapes attached to the outboard leading edge, ahead of the ailerons. Placed on the chord ahead of the normal laminar separation point, the triangles caused the flow to become turbulent and reattach, preventing a spanwise, continuous separation bubble from forming and thus from bursting. This dramatically reduced roll-off at stall.

Figure 15
Abrupt Stall

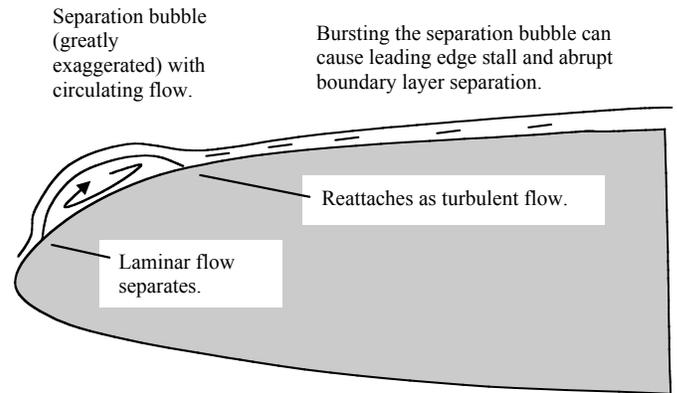
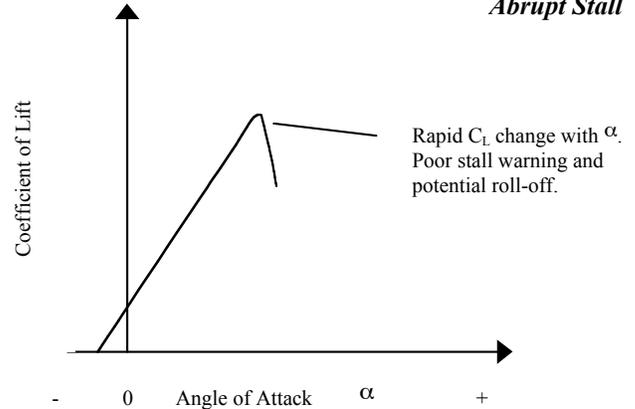
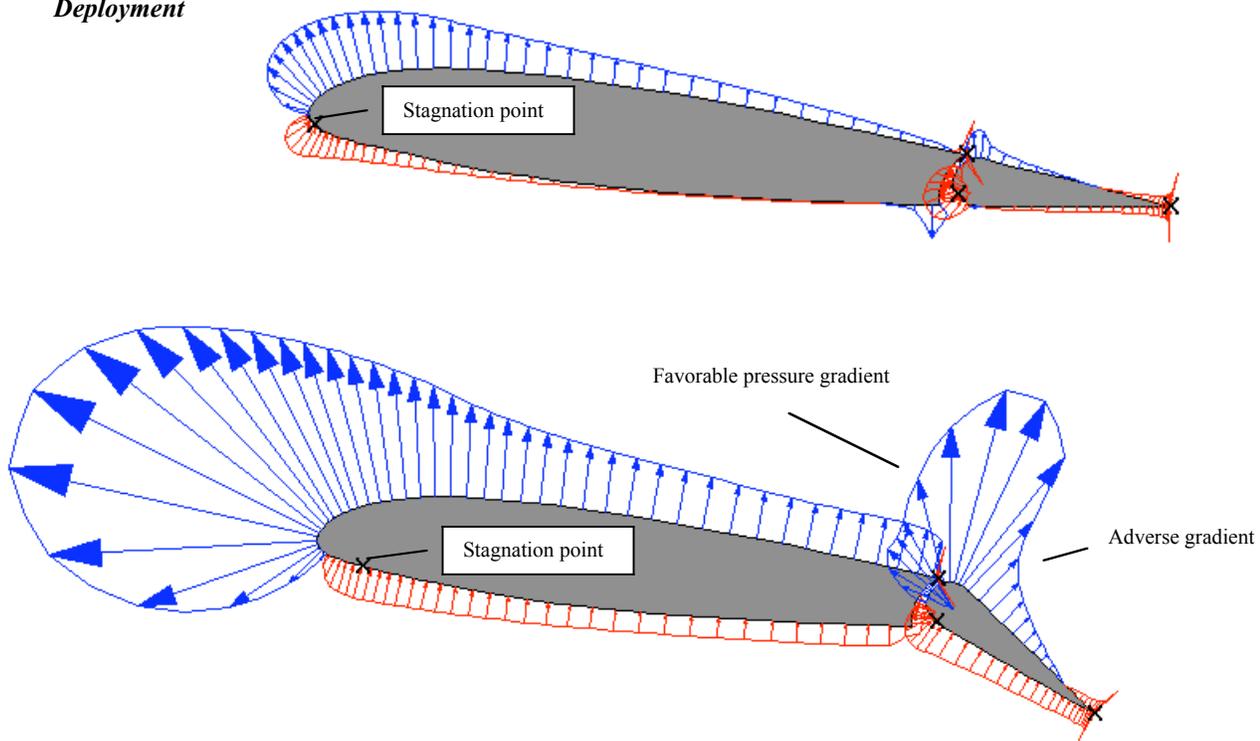


Figure 16
Flap
Deployment



Flaps

When you start to lower the flaps on our trainers, you'll see the tufts begin to show airflow separation along the trailing edge. The flaps increase local wing camber, which adds a second low-pressure peak (as shown in Figure 16) and a steep adverse pressure gradient at the trailing edge. If you increase the flap deflection further, or increase angle of attack, the adverse gradient over the flap becomes more severe. Flow reversal and separation occur and the tufts start dancing.

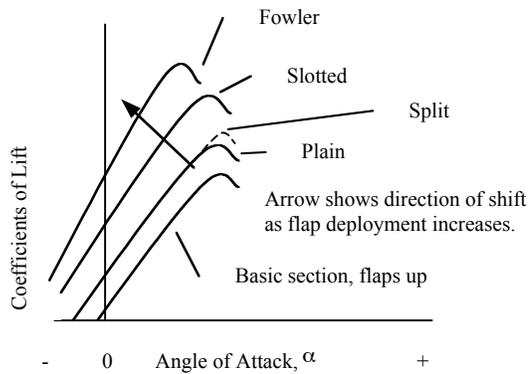
By studying Figure 16 you'll gain some insight into how flaps (and ailerons and rudders) work. Fundamentally, they modify lift by changing the velocity of the airflow. Notice how the increase in camber over the rear of the airfoil affects the overall pressure pattern. Camber increases local flow velocity, which decreases local surface static pressure. The result is a more favorable pressure gradient immediately *forward* of the deflected surface, which in turn causes air to accelerate over the wing ahead, resulting ultimately in lower surface pressures at the leading edge. At the same time, airflow below

the wing slow down, resulting in higher pressures there. (Notice the rearward shift in stagnation point that follows flap deployment. The flap-induced increase in leading-edge suction pulls more air forward from beneath the wing, sending the stagnation point further aft.)

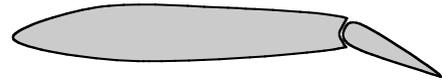
Figure 17 shows how the lift curve changes with flaps of different types. The Fowler flap is the most efficient because it produces the greatest increment in lift with the least increment in drag. Flaps shift the lift curve up and to the left as you increase the deflection angle. The shift is the result of the increase in camber. With the exception of the Fowler flap's curve, which becomes steeper, the slope of the lift curve remains unchanged with flap deployment. Maximum C_L increases with flaps, but occurs at a lower angle of attack than when the flaps are up. The zero-lift angle of attack becomes more negative (nose down).

Two-Dimensional Aerodynamics

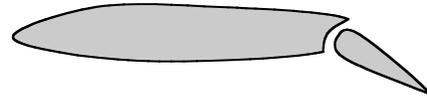
Figure 17
Lift Curve shift
with Flaps



Plain Flap



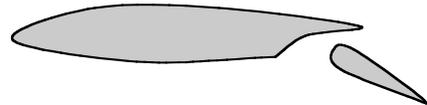
Slotted Flap



Split Flap



Fowler Flap



Because it reaches a higher maximum coefficient of lift, C_{Lmax} , with flaps extended, the wing will stall at a slower speed for a given aircraft weight. The benefit is disproportional, however, because large increases in C_{Lmax} are necessary to gain any substantial decrease in stall speed. For example, a 50 percent increase in C_{Lmax} produces only an 18 percent decrease in stall speed. A 100 percent increase in C_{Lmax} reduces stall speed by 30 percent, but at the expense of a large increase in drag.

Any type of flap is less effective on a thin wing than on a wing of greater thickness. Flaps are also less effective on swept wings compared to straight wings when their hinge line follows the sweep angle. You'll often see that the flaps themselves are not swept on an otherwise swept wing.

Circulation

Airflow passing over the top of a wing speeds up, while airflow passing beneath the wing slows down. These changes in velocity produce pressure differences between top and bottom, and thus lift.

Figure 18 shows what happens when you subtract the freestream flow from the accelerated flow over the top of the wing and from the decelerated flow beneath. The result reveals an embedded circulatory flow. (The circulation has a positive value above the wing; a negative value beneath.)

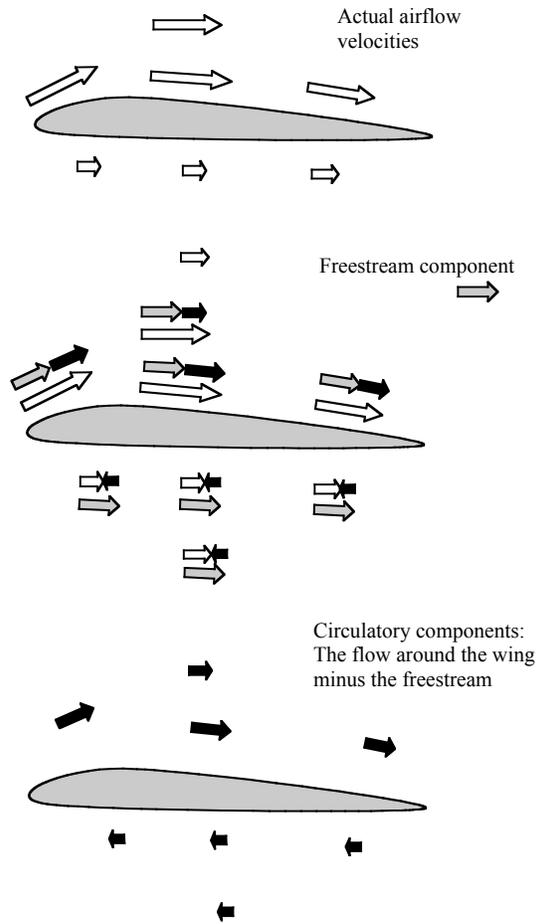
Circulation of this type doesn't mean that individual air particles actually travel completely around the wing, only that a circulatory tendency about the wing exists at any moment. The circulation is outside the boundary layer and extends well above and below the wing.

Although the idea of circulation pre-dates the Wright brother's first powered flight (the brothers weren't aware of it) and has great mathematical use, it's never been popular as a means of explaining lift to pilots. Imagining circulation around a wing isn't easy. It's easier to think of airfoils in wind-tunnel terms, the air moving in streamlines past a fixed airfoil that accelerates the air by means of an easily visualized venturi effect. In terms of the forces and moments produced, it doesn't matter whether the air or the wing moves. But in actual flying it's of course the wing that does the traveling. At subsonic speeds, it telegraphs its approach by the pressure wave it sends ahead, and the flow field starts accelerating or decelerating even before the wing arrives. Try to shift your frame of reference and to think in terms of the effects that the wing carries along with it, as it whizzes by, and of the effects it leaves behind in originally stationary air.

Circulation depends on airfoil design (leading edge radius, maximum thickness and its location, maximum camber and its location). Circulation increases with angle of attack up to the stall, and also with wing camber as modified by flaps. An aileron deflected down increases circulation over the affected part of the wing. One deflected up reduces circulation.

Other factors remaining equal, the greater the circulation the greater the velocity difference

Figure 18
Circulation



above and below the wing, and thus the greater the pressure difference and the resulting lift.

The circulation needed to produce lift of a given value increases as airspeed or air density *decreases*. The pertinent formula, in plain English, is:

$$\text{Lift} = \text{Density} \times \text{Freestream Velocity} \times \text{Circulation} \times \text{Span}$$

Bound Vortex/Tip Vortex

The circulation around an airfoil is called the bound vortex (Figure 19). On an actual three-dimensional wing, as opposed to a two-dimensional section, the bound vortex in effect turns the corner and becomes a trailing, tip vortex. The tip vortex is no longer an embedded flow bound to and carried along by the coordinates of the wing, but now a “free” or “true” vortex that remains attached to the same fluid particles and continues circulating long after the wing is gone. Because the tip vortex is generated by the pressure difference between the top of the wing and the bottom, and by the tendency of the airflow to try to even out that difference by “leaking” around the wingtip, ***tip vortex strength is a function of circulation.***

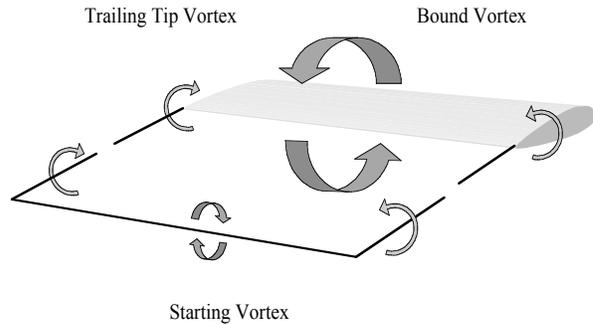
At low speeds and high angles of attack, when circulation necessarily rises, tip vortices increase in intensity. The heavier the aircraft and therefore the greater the required lift (lift equals weight in steady flight) the stronger the circulation has to be at a given airspeed. Consequently, heavy airplanes flying at slow approach speeds produce the most dangerous tip vortices. One reason to describe circulation is to give pilots a better sense of how potentially dangerous tip vortices are generated, and how their strength depends on the components of the lift formula:

$$\text{Lift} = \text{Density} \times \text{Freestream Velocity} \times \text{Circulation} \times \text{Span}$$

As the formula indicates, the circulation needed to generate lift equal to aircraft weight is also a function of wingspan. The longer the span the less circulation required. Consequently, at the same aircraft weight, long wings produce less intense tip vortices than short wings. This in turn lowers induced drag, as we’ll see.

In principle, a vortex can’t suddenly terminate in mid air. It has to form a continuous enclosed loop. Figure 19 shows the starting vortex that completes the loop. Figure 20 shows the role the starting vortex plays in getting circulation going.

Figure 19
Vortex System

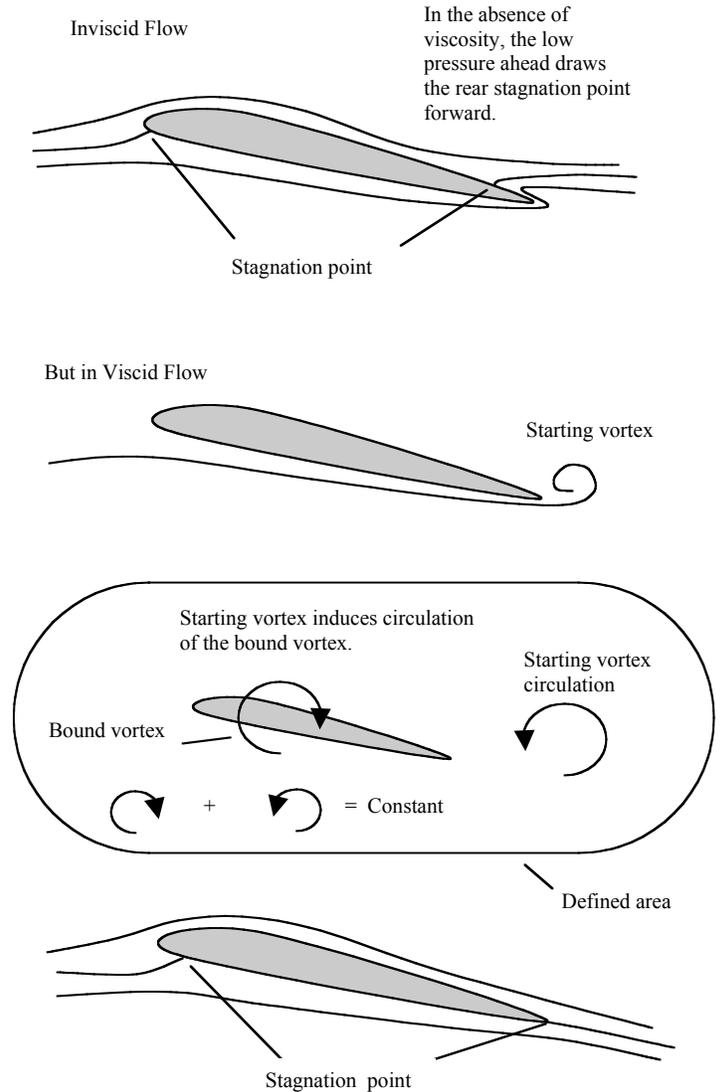


The Kutta Condition

The top of Figure 20 shows the theoretical streamline flow around an airfoil that would occur in the absence of friction and the resulting viscosity. The streamlines coming out from beneath the wing reverse direction. The rear stagnation point lies on the top of the wing, forward of the trailing edge.

That can't happen in nature, however, because the viscosity of the air prevents it from making the necessary sharp turn. Instead, as a wing begins to move forward from a standstill, a starting vortex forms. Because nature also says that the total circulation within any arbitrarily defined area must remain constant, an opposite vortex begins to form, the bound vortex. The wing begins to develop the circulation described earlier, and the airflow leaves the trailing edge smoothly, as shown in the bottom illustration. The starting vortex is left behind. This smooth departure is known as the Kutta condition. The upshot is that as angle of attack, camber, or airspeed change, the wing develops whatever circulation is necessary to maintain the Kutta condition. An increase in angle of attack, for example, causes a new starting vortex to form and be left behind. If angle of attack is decreased, a stopping vortex of opposite sign is formed and shed. A stopping vortex forms if the aircraft accelerates (since circulation required for lift goes down as airspeed increases). Ultimately, the vortices left behind break down due to friction and turbulence.

Figure 20
Kutta
Condition



Real viscid airflow requires meeting the Kutta condition. The circulation generated causes the front and rear stagnation points to shift as shown.