A Stroll down Kármán Street

Raghuraman N Govardhan and O N Ramesh

The Kármán vortex street is an exquisite flow pattern that can be seen in a variety of flow situations. In this article, we present some of the main features of the phenomenon starting from its interesting history, and also show why it is important in a number of engineering situations.

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If you do take a stroll down the so-called Kármán street as the title perhaps entices you to, you would be flanked by series of counter-rotating whirlpools on either side of the street which are dancing out of step with each other. No, this is no ordinary street for pedestrians but a promenade for whirlpools or vortices. It is in fact an exquisite flow pattern that occurs in a variety of situations involving bluff bodies. You might imagine a bluff body to be one with a cross sectional shape that offers a large resistance to the oncoming flow and retards it. The fluid stream fails to stick to the shape of the bluff body over which it flows, but breaks off behind it resulting in what is known as the wake region. And it is this wake that constitutes the stage for the dramatic fluid dynamic phenomenon known as the Kármán vortex street.

The phenomenon is so ubiquitous in flow behind bodies that it can be seen over a large range of length scales ranging from flow around strings of a musical instrument such as an Aeolian harp measuring about a millimetre, to terrestrial phenomena like flow past islands, with typical length scales of the order of a few hundred kilometres. This pervasiveness combined with its beauty and simplicity has resulted in it being one of the most visualised and familiar of all flow phenomena – a vividness that is equally striking to the specialist and the uninitiated alike.

What Exactly is a Kármán Street ?

Keywords
Karman, Karman vortex street.

Consider a stream of fluid flow past a circular cylinder, which

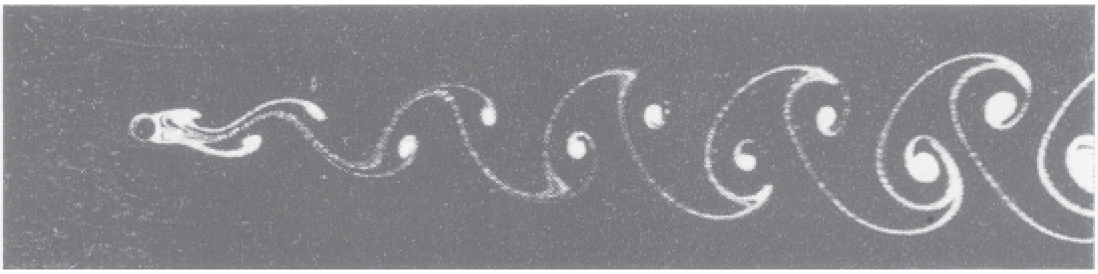

Figure 1. Photograph of the Kármán street (Reproduced from G K Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, 1967.)

can be called the archetypal or quintessential bluff body – a cross-section of 'perfect platonic circle' as R Narasimha aptly puts it in his two-part article. In the wake of the cylinder, the flow pattern consists of an alternating system of vortices or regions of rotation, as shown in *Figure* 1. A clockwise rotating eddy or vortex is 'shed' from the top side, followed by an anti-clockwise rotating vortex from the bottom side, and this process continues leading to a staggered system of alternating vortices. It is important to note that even if the oncoming flow is steady (i.e, does not change with time), the unsteadiness is intrinsically generated and the Kármán street is formed downstream of the body. To convey the pervasiveness of the pattern, a satellite image of the vortex shedding phenomenon behind an island is shown in *Figure* 2, the length scale of which is about 1,000,000 times larger than the laboratory phenomenon shown in *Figure* 1!

In order to convince you of the Kármán street's beauty and simplicity, we urge you to try the simple experiments described in *Box* 1. It will take you less than 30 minutes to see why fluid dynamicists are so enamoured by the Kármán street!

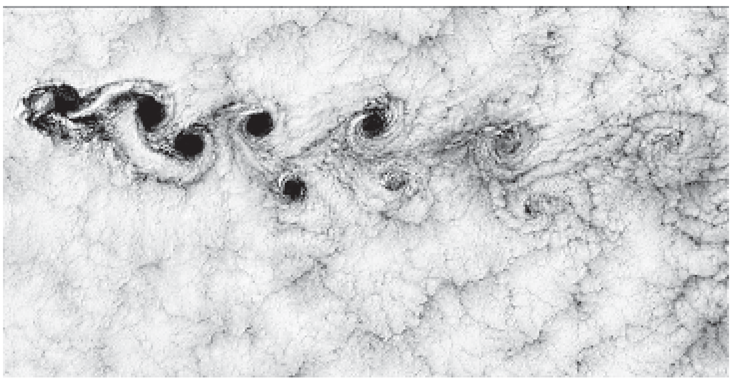

Figure 2. NASA satellite picture taken by Landsat 7 of the clouds around Alexander Selkirk Island in the southern Pacific Ocean. Amazingly even at this large terrestrial scale, a Kármán vortex street is formed as the wind-driven clouds flow over the island that rises sharply into the clouds. (Picture taken from http://landsat.gsfc.nasa.gov/earthasart/vortices.html)

Box 1. Visualize the Kármán Vortex Street in your Own 'Lab'

In order to 'construct' your lab, you will need a large bucket (preferably of a dark colour), a bottle of blue ink, and some talcum powder. Once you collect these essential elements, fill up the bucket with water and add reasonable amount of ink to it – maybe a cap full. Mix it well, so the water is uniformly blue. Now tap the talcum powder container with your fingers, so that the powder falls gently on to the water surface. Continue adding the powder until it covers the entire surface uniformly. With this the 'lab' is ready, and it is time to proceed to the experiment.

Take a cylindrical object like a pen, and pierce the surface of the water with it so that about 10 cm of the pen is submerged (If the powder runs away from the pen, you need to clean the pen surface properly). Keeping the pen perpendicular to the surface, gently move the pen along the surface and look at its wake. You should see the Kármán vortex street! This visualization technique is essentially similar to the one used by Prandtl. Play with the speed and size of the cylindrical body. You will see that the faster you move, the faster the vortices are generated, and the larger the body size, the bigger the eddies. Look closely and observe the vortices coming off the body carefully. If you are a little adventurous, you can even work out the Strouhal number and compare it to, say, equations (1) or (2)!

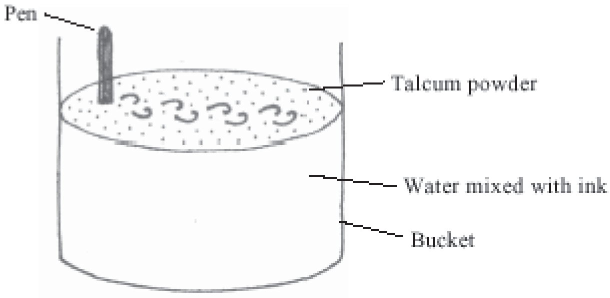

Figure A: Schematic showing components to visualize the Kármán street in your 'lab'.

History

Perhaps due to the simplicity of the flow situation, the Kármán street is readily observed and has been recorded even before the Renaissance period, when the modern scientific thought blossomed in Europe. The earliest recorded observation of the Kármán street appears to be a painting from the 14th century depicted in a church in Bologna, Italy, that shows St. Christopher, carrying Infant Jesus in his arms, wading through a stream of water. The painting shows a series of alternating vortices coming off St. Christopher's legs. Apparently, this painting was one of the inspirations for von Kármán who initiated the formal scientific study of the vortex-shedding phenomenon. He points

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Kármán

out in his autobiography in reference to the picture of St.Christopher in Bologna: "*The problem for historians may have been why Christopher was carrying Jesus through the water. For me it was why the vortices.*"

Although the vortex street had been observed in a variety of situations for centuries before the last, the philosophical implications of such a flow configuration must have run counter to the common run of thinking and conventional wisdom in the early part of the 20th century. The prevalent idea of the time seems to be that steadiness of geometry and oncoming flow must necessarily result in steadiness downstream. (This expectation is of course not true in turbulent flows characterized by high velocities or more precisely high Reynolds numbers. For this case, nominally steady boundary conditions result in an apparently chaotic flow, but then turbulence is a different kettle of fish altogether.) In view of this, the phenomenon of vortex shedding was a source of intrigue and seems to have been considered even counter-intuitive in the early part of the 20th century, as the following account illustrates.

In 1911, the celebrated German fluid dynamicist Ludwig Prandtl was investigating the static pressure distribution along the surface of a circular cylinder when it was placed in a steady stream of water. Hiemenz, a doctoral candidate, was given the task of making the measurements. Hiemenz was very frustrated to repeatedly find unsteady fluctuations in the channel. When Hiemenz reported this to Prandtl, he was told that the cylinder was probably not circular. Despite spending considerable effort in polishing and smoothing the cylinder by employing the famed German fastidiousness for precision, Hiemenz still could not get rid of the flow oscillation. When he reported this to Prandtl, he was told that his channel was probably not symmetrical! Hiemenz started perfecting the channel. Kármán, who at the time was a graduate assistant working with Prandtl, would every morning religiously ask Hiemenz, "Herr Hiemenz, is the flow steady now?", and a crestfallen Hiemenz used to sadly reply, "it always oscillates". Kármán eventually figured out that the unsteady phenomenon must be intrinsic to the flow. Thoroughly absorbed by the problem, over a weekend, he took it upon himself to calculate the stability of a system of vortices. He showed that the observed vortex system of staggered asymmetrical vortices was the only stable system. And it is only a stable configuration that one gets to observe in practice. This contribution, which was presented to the Göttingen Academy by Prandtl and a subsequent paper, are in fact Kármán's contribution to this phenomenon. Over time, this contribution of Kármán (explained in more detail in *Box* 2) resulted in the pattern being called the Kármán Vortex Street. If you are a research student, you could take heart in the thought that the next time you complain to your research supervisor that the experiment does not work as 'expected' despite endless tinkering, you may be right and might well be on your way to a major discovery!

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To set the historical record straight, it is important to mention here that the vortex street had been seen and photographed prior to Kármán (in the scientific world) by a French professor named Henri Benard, and although it is usually referred to as the Kármán vortex street, the French refer to it as the Benard–Kármán vortex street.

What is the Frequency at which Vortices or Eddies Peel off the Body?

The above question turns out to be the crux of bluff body aerodynamics and a very central issue in a number of engineering applications. Clearly, the most dangerous aspect of the Kármán vortex street is its periodic nature. It leads to possibilities of resonance with natural structural modes of flexible bodies, and hence results in failure of structures that may otherwise have been safe. The infamous Tacoma Narrows Bridge disaster in USA in 1940 is an example where the culprit was thought to be the structural vibration and resonance induced due to Kármán vortex shedding phenomenon. A detailed account of this episode is given in Kármán's own words in the Classics section of this issue.

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Box 2. Kármán 's Contribution to the Kármán Vortex Street

Kármán himself best summarizes his contribution in this excerpt from *Aerodynamics: Selected topics in the light of their historical development*. (Theodore von Kármán, Cornell University Press, 1954)

"What I really contributed to the aerodynamic knowledge of the observed phenomenon is twofold. I think I was the first to show that the symmetric arrangement of vortices (*Figure* B, upper), which would be an obvious possibility to replace the vortex sheet, is unstable. I found that only the anti-symmetric arrangement (*Figure* B, lower) could be stable, and only for a certain ratio of the distance between the rows and the distance between two consecutive vortices of each row. Also, I connected the momentum carried by the vortex system with the drag and showed how the creation of such a vortex system can represent the mechanism of the wake drag…"

The first contribution of Kármán resulted in his prediction that the only arrangement of vortices that was not unstable was an anti-symmetric arrangement of vortices, with spacing ratio, $b/a = 0.28056$, where a = inter-vortex spacing in one row and b = distance between vortex rows. All other arrangements were shown to be unstable. Subsequent experiments at Göttingen and other places, found that the ratio of the distances seen in actual experiments was quite close to the ratio calculated by Kármán. In fact, the Kármán vortex street is amazingly stable, and it is possible to see a large number of vortices downstream of the body. A hint of this is seen in *Figure* 1, although it is perhaps best illustrated by the picture that one Prof Wille presented to Kármán on his 75th birthday with 75 shed vortices, with the engaging caption: *Ad Infinitum*.

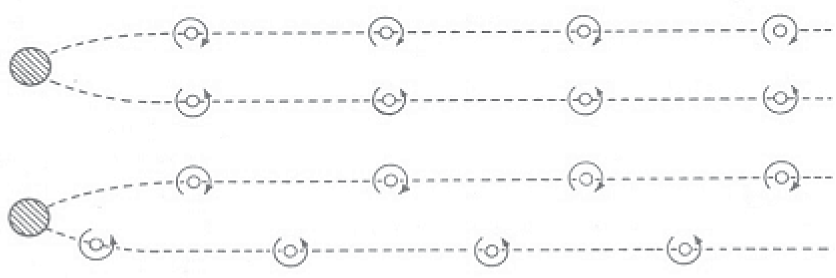

Figure B. Double rows of alternating vortices; symmetric (upper) and asymmetric (lower) arrangements. (Reproduced from Theodore von Kármán, Aerodynamics: Selected topics in the light of their historical development, Cornell University Press, 1954.)

Strouhal, in 1878, made the first measurements of the frequencies associated with the shedding of vortices, well before the scientific community had come to grips with the fluid dynamics of the Kármán vortex street. Strouhal measured the frequency of acoustic tones coming from a vertical wire that was made to revolve at uniform speed about a parallel axis, and showed that the origin of these acoustic tones is essentially aerodynamic. By varying the diameter (D) of the wire and the speed (U) of the wire motion, Strouhal found that (within certain limits) the acoustic frequency (f) was expressible as

$$f = 0.185\ U/D. \tag{1}$$

For a large range of velocities and body shapes, as a rule of thumb, the Strouhal number is approximately 0.2.

In other words, the acoustic frequency is related directly to the periodic formation of vortices – a remarkable result that bridges the gap between two seemingly different fields of mechanics and acoustics. The above relation also suggested that the non-dimensional number (fD/U) was approximately a constant (within the range of values) and has since become known as the Strouhal number: $S = fD/U$. It turns out that for a large range of velocities and body shapes, as a rule of thumb, the Strouhal number is approximately 0.2.

That the frequency should vary directly with velocity and inversely with diameter can be physically understood as follows. If the velocity is large, a large number of vortices cross a given location in unit time. Likewise, the larger the diameter the longer it is going to take for a vortex to be formed and peel off from the body. In the light of the last statement, one can have a rough picture of the vortex formation mechanism before it is shed from the body. When the fluid stream reaches roughly the topmost point of the cylinder, it finds itself unable to negotiate the rear half of the cylinder where there is an uphill pressure gradient. Hence the fluid separates from the top surface, constitutes itself into a clockwise swirling vortex as it approaches the rear end of the cylinder after which it peels off as a shed vortex. Hence the vortex formation time is proportional to the perimeter (or the diameter) of the cylinder.

Lord Rayleigh subsequently evaluated the data of Strouhal in the light of dimensional analysis. He made the observation that the frequency of vortex shedding depended primarily on the velocity of the wind (U), the diameter (D) of the wire, and the

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kinematic viscosity of the fluid (ν). Consideration of viscosity in the relation was necessary to explain the deviations of some measurements of Strouhal from equation (1), when the flow speed for example was varied over a much larger range, and was also consistent with some temperature effects on frequency that Strouhal had observed. In 1915, based on similarity arguments and the data of Strouhal, Lord Rayleigh obtained a formula for the frequency:

$$fD/U = 0.195\ (\ 1 - 20.1\nu/UD), \qquad (2)$$

where fD/U is the Strouhal number (S) and UD/ν is the Reynolds number (Re).

The ingenious experimental method of Strouhal had one drawback though, as noted by Rayleigh. As the wire continually revolved, there was the complication that the air was set into motion by the revolving parts of the apparatus, and the fluid ahead of the wire was not at rest as presumed. More than 70 years later, these concerns were addressed in the classic wind-tunnel measurements of Roshko in 1954, who measured the frequencies using a hot-wire velocity probe. For the low Reynolds number laminar region, Roshko condensed his results to an equation of the form

$$S = 0.212\ (1 - 21.2\ /\ Re), \qquad (3)$$

which is really close to Rayleigh's formula (2) based on Strouhal's experiments of 1878!

Subsequently there have been more advances in the understanding of the relationship between Strouhal and Reynolds number. One of the principal features of the more recent understanding is that three-dimensionalities of the flow are crucial, even when considering flow past a nominally two-dimensional body like a long circular cylinder. In order to picture this clearly, flow visualization in the plane containing the cylinder axis is shown in *Figure* 3. In this plane, the Kármán vortices are seen as long lines that are coming off all along the length of the cylinder, as in

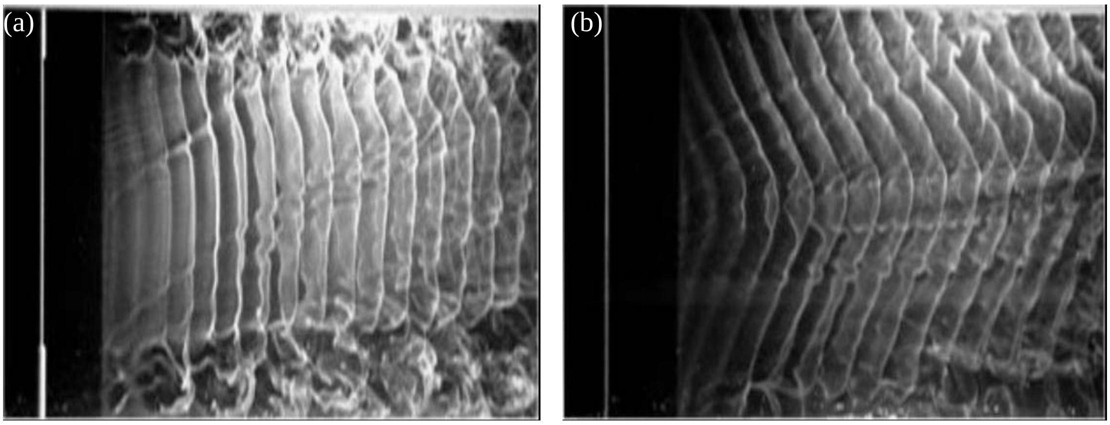

Figure 3. Flow visualization in the plane containing the cylinder axis. In (a), the long vortices are shed parallel to the cylinder axis – parallel vortex shedding, while in (b), the vortices are shed at an oblique angle to the cylinder axis – oblique vortex shedding. The cylinder is the 'vertical line' at the extreme left in both images. (Reproduced from MSc (Engg) thesis of R S Chopde, IISc, Bangalore.)

Figure 3(a). (The picture of the staggered series of Kármán vortices shown in *Figure* 1 is seen in a cut perpendicular to the cylinder axis.) As shown in the 1980s, these long vortices can be induced to shed either parallel to the cylinder axis, as in *Figure* 3(a), or at an oblique angle (θ) to the cylinder axis, as in *Figure* 3(b). When frequency measurements are performed in the wake for the two cases, it is found that there are different frequencies associated with each. This dependence of frequency and Strouhal number on shedding angle (θ) had in fact led to significant scatter in the measurements of S-Re and this was a source of some lively debates in the literature in the 1950s and 60s. By taking into account the obliqueness angle appropriately, Williamson, in 1988, showed that the frequency measurements for low Reynolds number flows can be collapsed on to a single curve of S vs. Re, using a simple relationship, thereby satisfactorily resolving the whole debate.

Vortex Shedding from 3-dimensional Bodies

When a geometrically three-dimensional bluff body, for example a sphere, is placed in a flow, one might expect periodic vortex formation as in the case of a two-dimensional body like a circular cylinder. However, since the body has a finite extent in the side view (see *Figure* 4), the vortices formed cannot be long structures as they are for the cylinder. Further, it can be shown that under normal conditions a vortex cannot end abruptly inside the flow – a vortex has to end at a solid boundary or

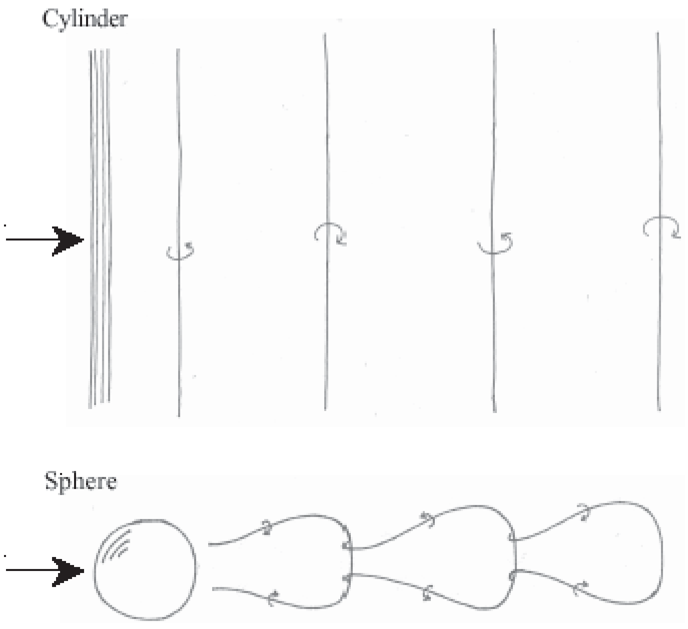

Figure 4. Schematic of the vortex shedding pattern in side view for a cylinder and a sphere. Unlike the cylinder case, for the sphere the vortices are inter-connected.

connect back to itself like in a vortex ring (Helmholtz vortex theorem). By combining the above two observations, we end up with the conclusion that there must be connections between the ends of the vortices for three-dimensional bodies. This is in fact what is observed in experiments for three-dimensional bodies such as a sphere, as shown by a schematic of the vortex pattern in *Figure* 4. These structures are referred to variously as "hairpin-shaped" structures and "interconnected loops" in the literature. A simple way of appreciating these wakes is to release a spherical drop of ink into a stationary tank, and to observe the resulting intricate patterns. The ink here acts as a marker for the vortex structures created by the 'spherical' drop falling in the tank. Try it out in a glass cup or any transparent tank – you will be amazed!

Vortex-Induced Vibrations: Oscillations Induced by Kármán Vortex Street

The vortices or eddies coming off a bluff body form alternately on either side of the body and are shed into the wake. One can see

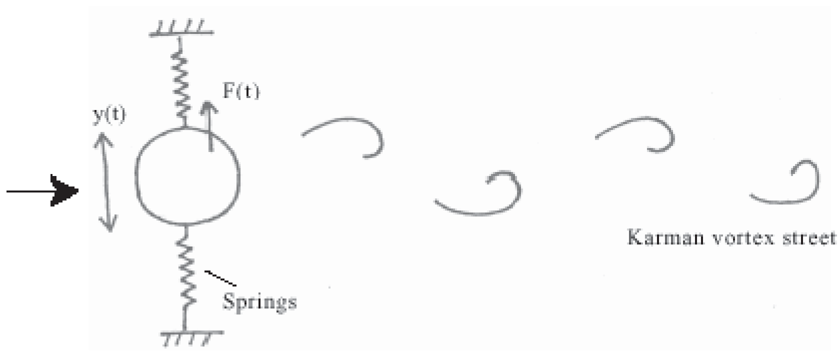

Figure 5. Schematic of an elastically-mounted cylinder in a fluid flow. In this case, the forcing due to vortex shedding, F(t), can induce body oscillations, y(t). Such oscillations caused by vortex shedding are referred to as vortex-induced oscillations, and can lead to very large body oscillations as in the Tacoma Narrows Bridge disaster.

that there is an instantaneous asymmetry in the flow close to the body as one of the vortices is formed and then shed. This naturally leads to a force perpendicular to the flow direction (say upwards), which in aerodynamics is usually referred to as the lift force. Subsequently, when the vortex of opposite sign is formed on the other side, there is again an asymmetry in the flow, but in this case we would expect the lift force to be exactly opposite (downwards). Hence, as the vortices are alternately formed and shed, the lift force continually fluctuates between positive and negative values. Actual measurements indicate that the lift force can be reasonably approximated by a sinusoid.

Now consider a situation where the body is itself flexibly mounted on springs, as illustrated in the sketch below. In this case, the body has a structural frequency that depends on the stiffness of the springs and the body mass (remember from your basic mechanics that the natural frequency of a spring-mass system is $(1/2\pi)\sqrt{k/m}$, where k is spring stiffness and m is the mass). Note that this structural frequency is independent of flow parameters. On the other hand, the vortex shedding frequency increases linearly with flow speed (from equation (1)). It is clear that at some flow speed the two frequencies will coincide leading to resonance with significant amplitudes of body oscillation.

Experiments in such configurations as in *Figure* 5 show that there are indeed large oscillations that can result when the

Peak-peak amplitudes of more than 3 diameters have been observed for such (as in Figure 5) elastically-mounted circular cylinders!

vortex shedding frequency is close to the structural frequency. Peak-peak amplitudes of more than 3 diameters have been observed for such elastically-mounted circular cylinders!

Vortex-induced vibrations of the type discussed above are however not easy to predict. The main reason for this is that the vortex shedding phenomenon is itself significantly altered by the body oscillations. Apart from quantitative changes in the lift force, visualizations have indicated that the resulting vortex pattern can be radically different compared to that from a stationary body. For example, instead of the normal '2 Single' vortices coming off each cycle (so called 2S mode) as in the Kármán street, there can be '2 Pairs' of counter-rotating vortices shed per oscillation cycle – so called 2P mode. This 2P mode has a different magnitude of force and phase relative to that of the body motion. The phase here is very crucial as it determines energy transfer between the fluid and the structure. Hence, close to resonance, as the body oscillation amplitude begins to increase from rest, the forcing due to vortex shedding changes quantitatively as well as qualitatively. This in turn affects the body oscillations. Therefore, predictions of the oscillation amplitudes are not easy. A number of studies over the past 30 years, and in particular over the last decade, have addressed this problem and shown the rich variety of possibilities with this conceptually simple system.

Studies of vortex-induced vibrations especially, for circular cylindrical shapes, have attracted considerable interest because of their widespread use in offshore platforms used in oil and gas production. Because of the long lengths of cylindrical tubes involved in these operations, these tubes are prone to vortex-induced oscillations. Imagine a 1-metre diameter tube carrying oil from the seabed to the platform subjected to vortex-induced vibrations and oscillating at 3 metres peak-peak amplitude! There is anecdotal evidence that the collapse of some offshore rigs is due to vortex-induced vibrations.

Conclusion

The Kármán vortex street is a ubiquitous flow pattern that is seen in a variety of flow situations. We have briefly given some of the salient points regarding this phenomenon starting from its interesting history. It consists of a staggered alternate system of vortices, which was shown to be the only stable possibility for such a system of vortices by Kármán.

On the practical side, the Kármán street can lead to unwanted noise or even cause failure of structures when the vortex shedding frequency coincides with the natural frequency of structure. This type of vortex-induced oscillations can occur in a variety of situations such as chimneys, bridges, heat exchanger tubes, overhead power cables and marine structures. Therefore understanding the physics of the Kármán vortex street is crucial to avoid disasters in these situations.

It is nearly a century since the pioneering work of Kármán, and over this period our understanding of the Kármán street has increased considerably. However, as Roshko recently stated in a review, much of this knowledge "remains almost entirely in the empirical, descriptive realm of knowledge."

Acknowledgements

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Suggested Reading

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[2] **Jaywant H Arakeri and P N Shankar, Ludwig Prandtl and Boundary Layers in Fluid Flow, *Resonance*, Vol.5, No.12, 2000.**

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